Teaching schedule

Session	Topics
***15 – 18**	**5. Gas power cycles** Basic considerations in the analysis of power cycle; Carnot cycle; Air standard cycle; Reciprocating engines; Otto cycle; Diesel cycle; Stirling cycle; Brayton cycle; Second – law analysis of gas power cycles
19 – 21	**6. Vapor and combined power cycles** Carnot vapor cycle; Rankine cycle; Deviation of actual vapor power cycles; Cogeneration; Combined gas – vapor power cycles
22 – 24	**7. Refrigeration cycles** Refrigerators and heat pumps; Reversed Carnot cycle; Ideal vapor – compression refrigeration cycle; Actual vapor compression refrigeration cycle

Objectives

- Evaluate the performance of gas power cycles for which the working fluid remains a gas throughout the entire cycle.
- Develop simplifying assumptions applicable to gas power cycles.
- Analyze both closed and open gas power cycles.
- Solve problems based on the Otto, Diesel, Stirling, and Ericsson cycles.
- Solve problems based on the Brayton cycle; the Brayton cycle with regeneration; and the Brayton cycle with intercooling, reheating, and regeneration.
- Perform second-law analysis of gas power cycles.

Gas power cycle

- Carnot cycle
- Otto cycle: ideal cycle for spark-ignition engines
- Diesel cycle: ideal cycle for compression-ignition engines
- Stirling and Ericsson cycles
- Brayton cycle: ideal cycle for gas turbine engine

If the Carnot cycle is the best possible cycle, why do we not use it as the model cycle for all the heat engine?

❑ **Analysis of power cycles**

- Neglect friction
- Neglect the required time for establishing the equilibrium state

Carnot Cycle

The Carnot cycle gives the most efficient heat engine that can operate between two fixed temperatures T_H and T_L; it is independent of the type of working fluid and can be closed or steady flow.

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

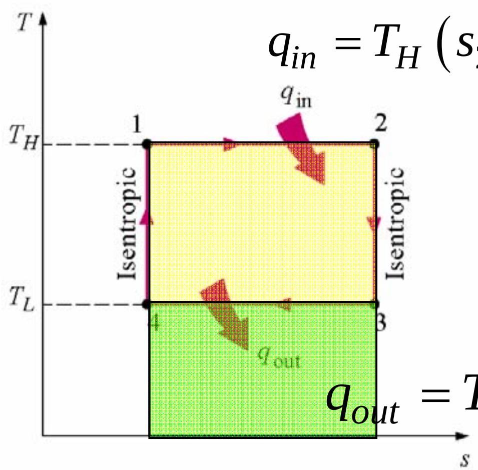
Carnot Cycle

Process	Description
1-2	Isothermal heat addition
2-3	Isentropic expansion
3-4	Isothermal heat rejection
4-1	Isentropic compression

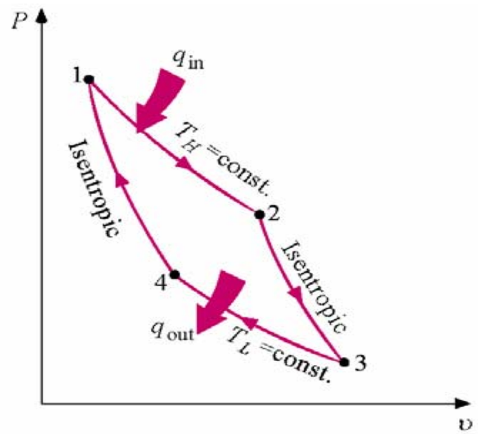
$$\eta_{th} = \frac{q_{in} - q_{out}}{q_{in}}$$

$$\eta_{th,\,Carnot} = 1 - \frac{T_L}{T_H}$$

$$q_{in} = T_H\left(s_2 - s_1\right)$$

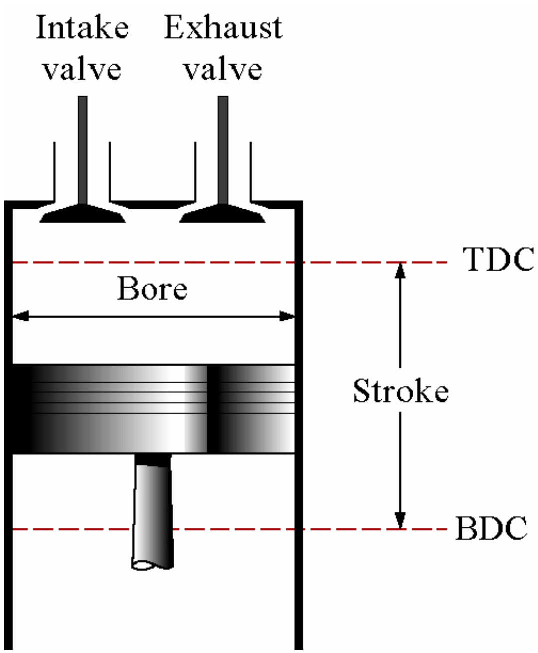


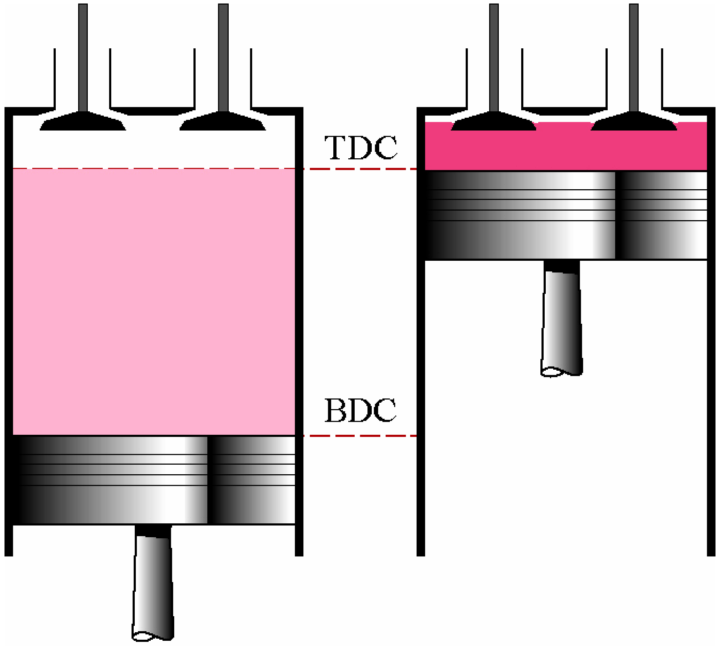
$$q_{out} = T_L\left(s_3 - s_4\right)$$



Area, which is enclosed by cyclic curve presents net work or heat transfer during the cycle.

Reciprocating engine





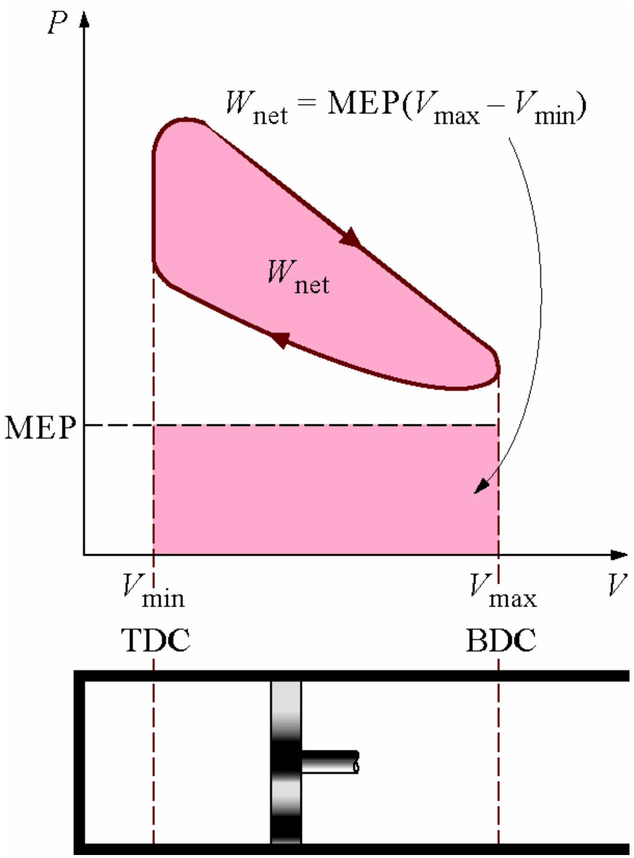
(*a*) Displacement volume

(*b*) Clearance volume

- TDC = Top Dead Center
- BDC = Bottom Dead Center
- Clearance volume = minimum volume formed in cylinder

Compression ratio, r = $$\frac{\text{Max. volume}}{\text{Min. volume}} = \frac{V_{BDC}}{V_{TDC}}$$

Reciprocating engine



$$\text{Work} = \vec{F} \cdot \vec{s}$$

$$W_{net} = \text{MEP} \times \text{Piston area} \times \text{Stroke}$$

- MEP = Mean Effective Pressure

$$W_{net} = \text{MEP} \times \text{Displacement volume}$$

$$\text{MEP} = \frac{W_{net}}{V_{max} - V_{min}}$$

Larger value of MEP gives more net work per cycle, thus perform better

Gas power cycles

❑ Analysis of power cycles

- The working fluid remains a gas throughout the entire cycle.
- Neglect friction
- Neglect the required time for establishing the equilibrium state

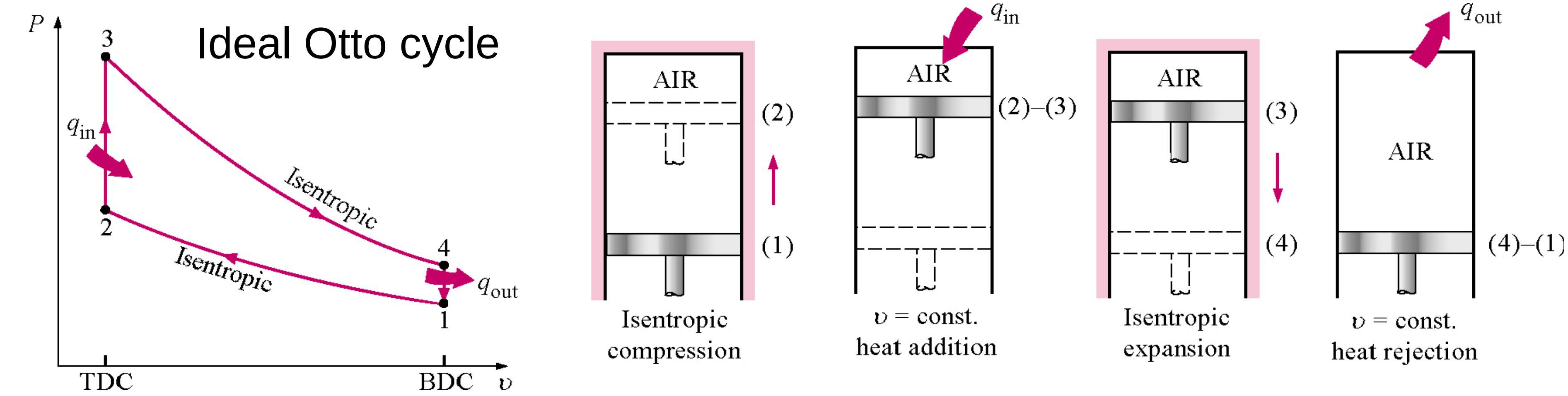
❑ Air-standard assumptions

- The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.
- All the processes that make up the cycle are internally reversible.
- The combustion process is replaced by a heat-addition process from an external source.
- The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.

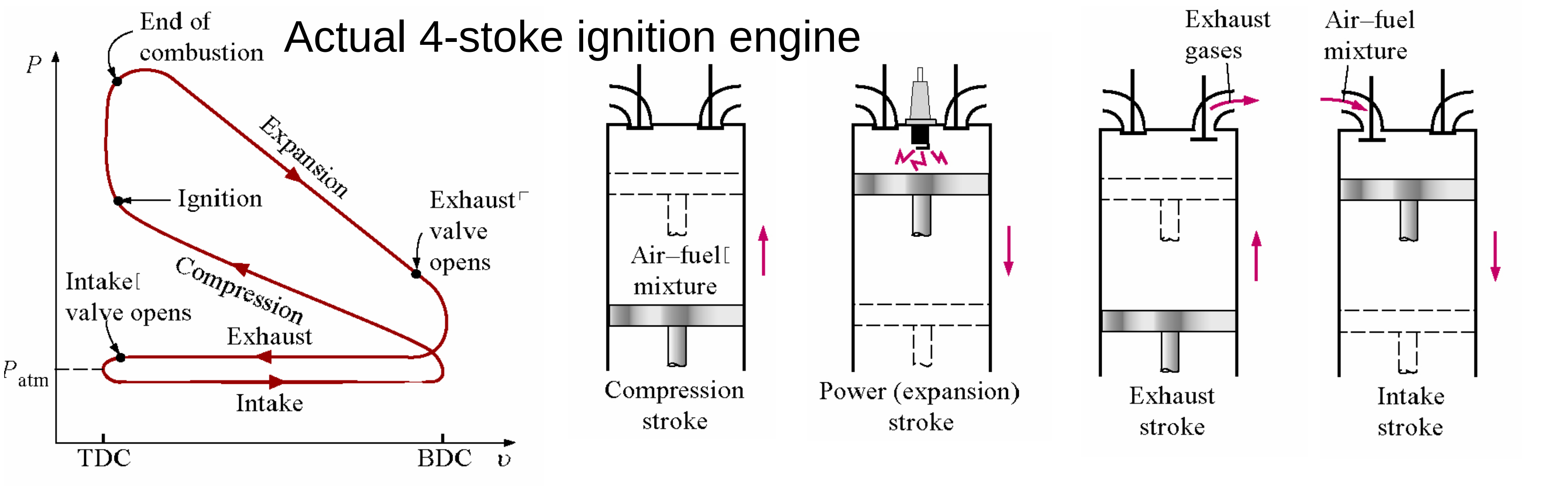
Otto cycle

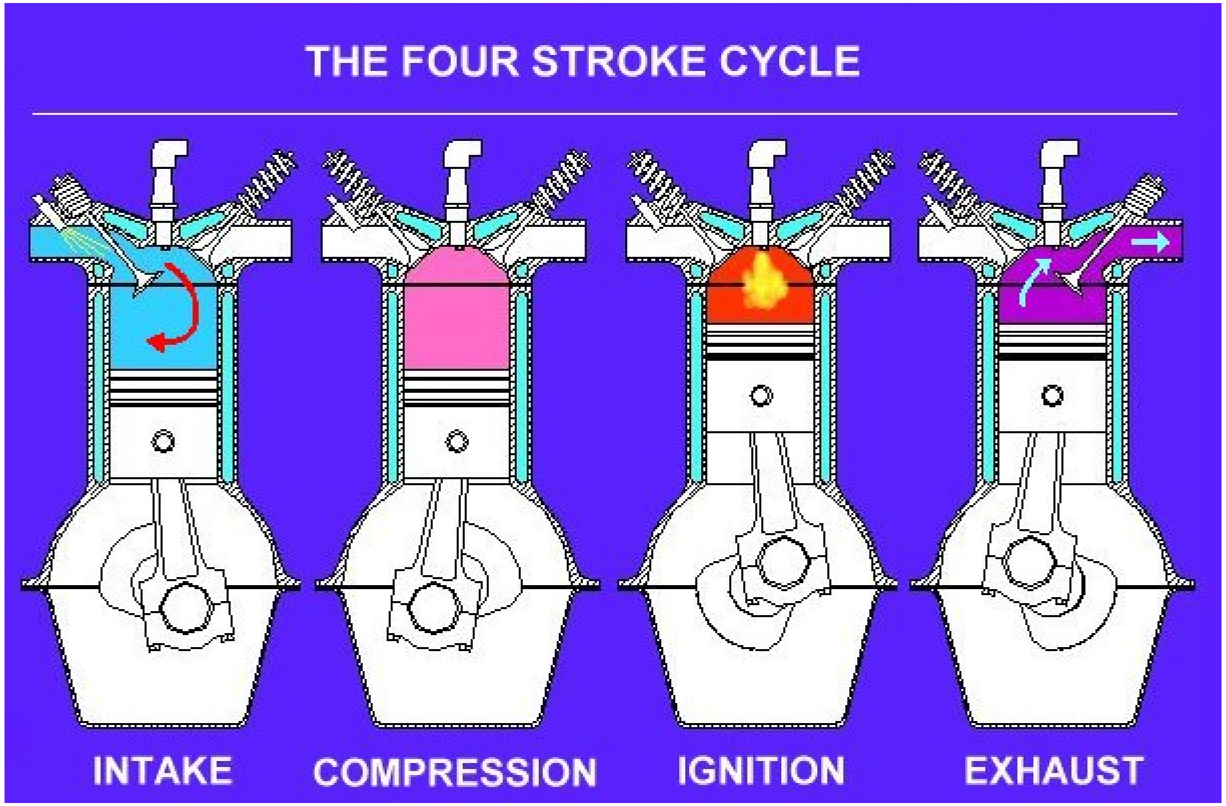
- An ideal cycle for spark-ignition engines
- Nikolaus A. Otto (1876) built a 4-stoke engine

Ideal Otto cycle

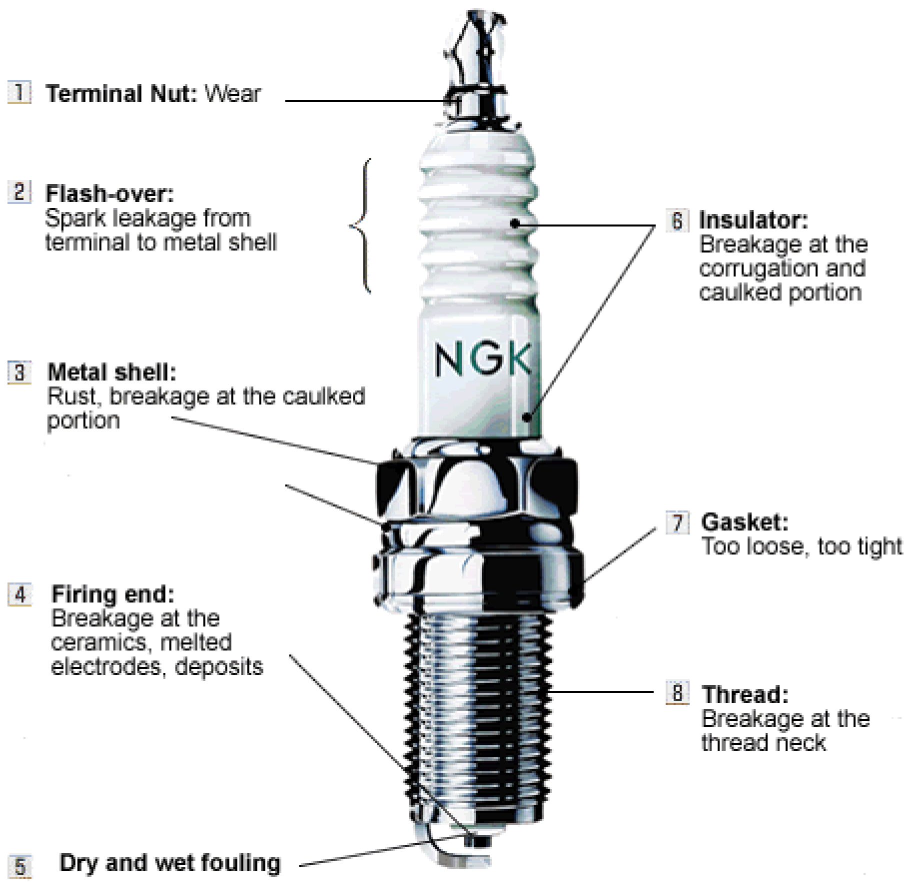


Actual 4-stoke ignition engine




THE FOUR STROKE CYCLE
INTAKE
COMPRESSION
IGNITION
EXHAUST

Spark plug

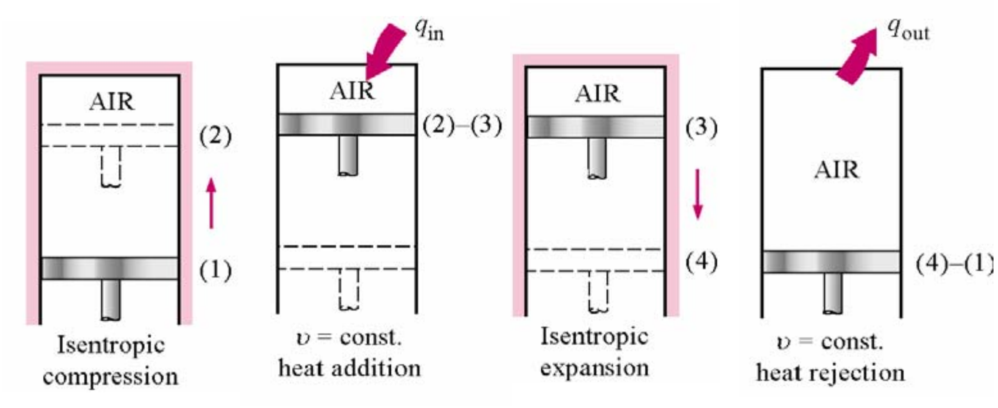


1 Terminal Nut: Wear
2 Flash-over:
Spark leakage from terminal to metal shell
3 Metal shell:
Rust, breakage at the caulked portion
4 Firing end:
Breakage at the ceramics, melted electrodes, deposits
5 Dry and wet fouling
6 Insulator:
Breakage at the corrugation and caulked portion
7 Gasket:
Too loose, too tight
8 Thread:
Breakage at the thread neck
NGK

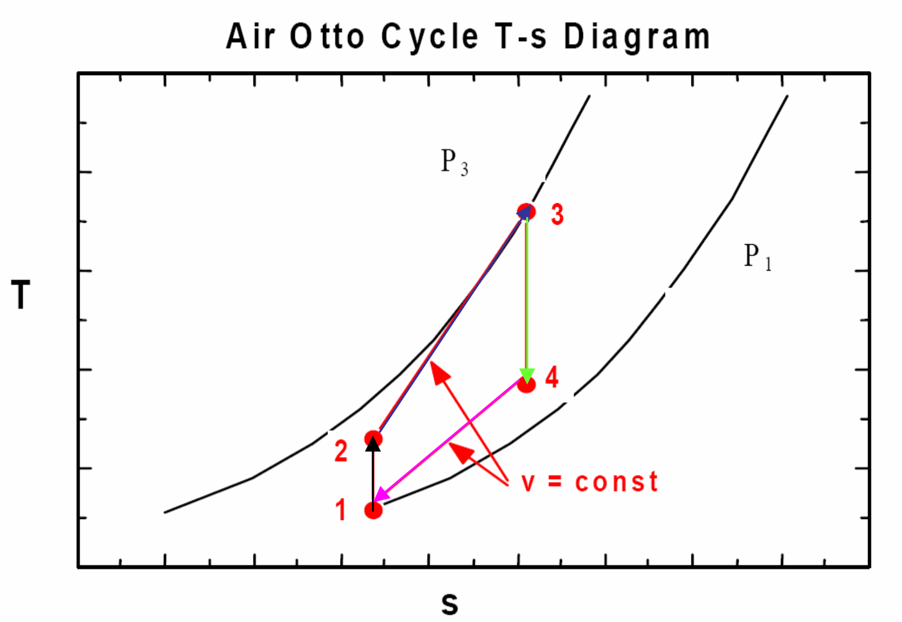
The air-standard Otto cycle

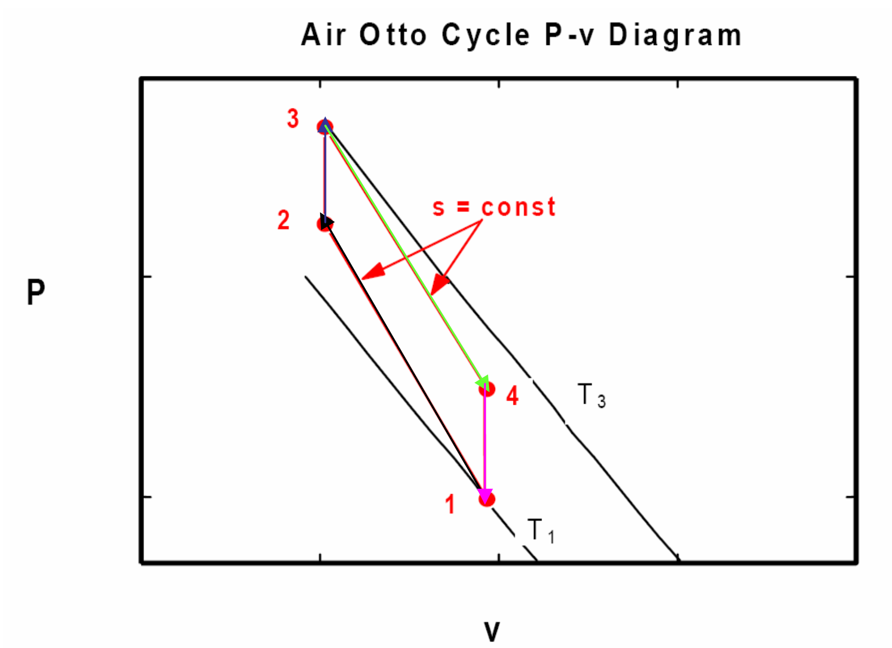
The air-standard Otto cycle is the ideal cycle that approximates the spark-ignition combustion engine.

Process	Description
1-2	Isentropic compression
2-3	Constant volume heat addition
3-4	Isentropic expansion
4-1	Constant volume heat rejection



The *T-s* and *P-v* diagrams are

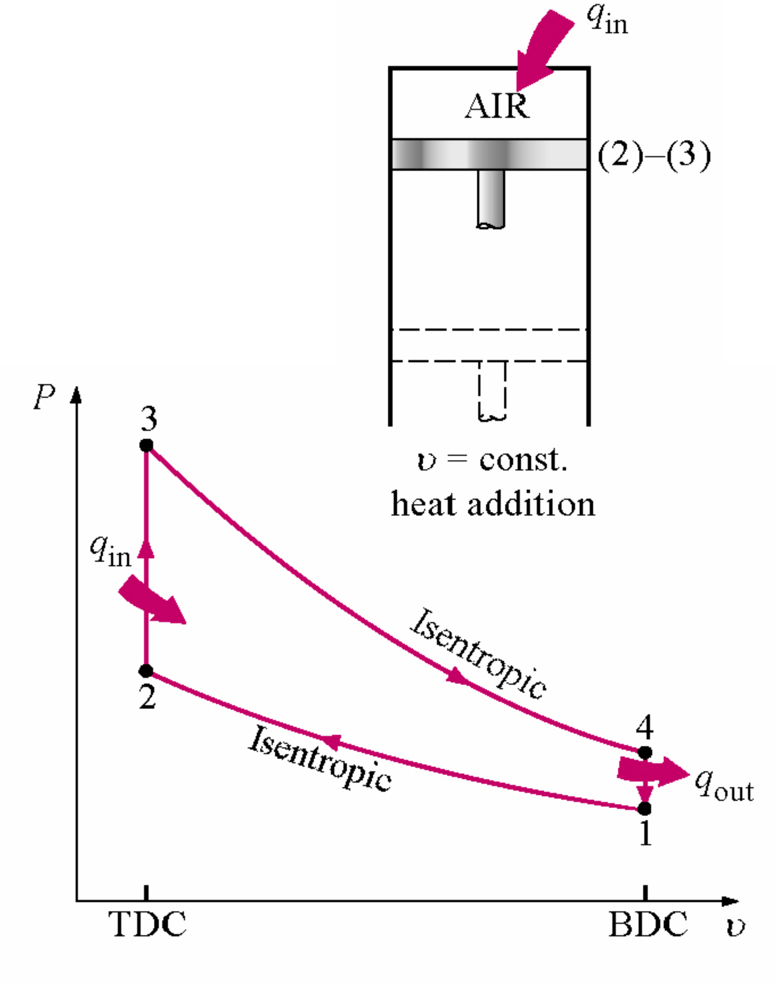




Thermal Efficiency of the Otto cycle:

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{Q_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Apply 1st law closed system to process 2-3, constant volume heat addition



$$E_{in,23} - E_{out,23} = \Delta E_{23}$$

$$Q_{net,23} - W_{net,23} = \Delta U_{23}$$

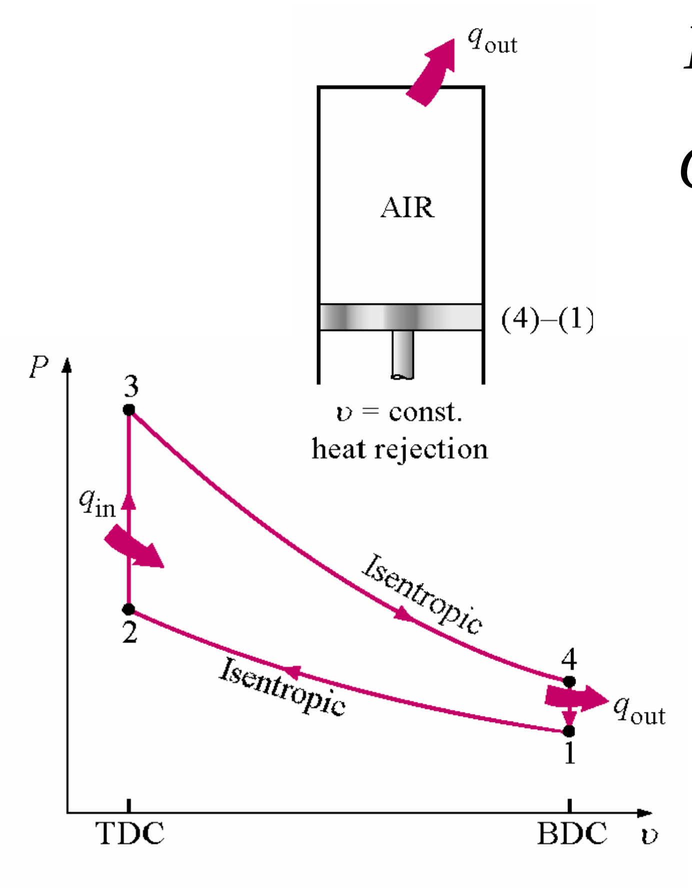
$$W_{net,23} = W_{other,23} + W_{b,23} = 0 + \int_2^3 P dV = 0$$

$$Q_{net,\,23} = \Delta U_{23}$$

Thus, for constant specific heats,

$$Q_{net,\,23} = Q_{in} = mC_v(T_3 - T_2)$$

Apply 1st law closed system to process 4-1, constant volume heat rejection



$$E_{in,41} - E_{out,41} = \Delta E_{41}$$

$$Q_{net,41} - W_{net,41} = \Delta U_{41}$$

$$W_{net,41} = W_{other,41} + W_{b,41} = 0 + \int_1^4 PdV = 0$$

Thus, for constant specific heats,

$$Q_{net,\,41} = \Delta U_{41}$$

$$Q_{net,\,41} = -Q_{out} = mC_v(T_1 - T_4)$$

$$Q_{out} = -mC_v(T_1 - T_4) = mC_v(T_4 - T_1)$$

The thermal efficiency becomes

$$\eta_{th,\,Otto} = 1 - \frac{Q_{out}}{Q_{in}}$$

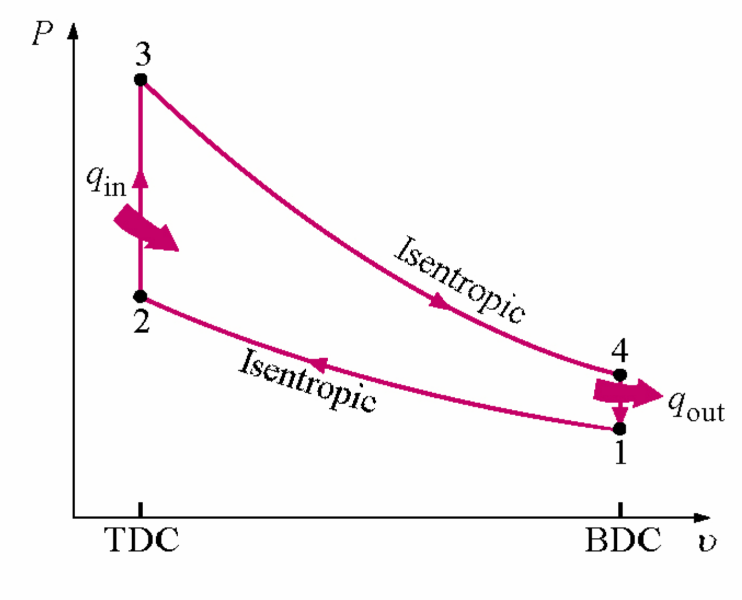
$$= 1 - \frac{mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)}$$

$\Rightarrow$

$$\eta_{th,\,Otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

Processes 1-2 and 3-4 are isentropic, so



$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad and \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1}$$

Since $V_3 = V_2$ and $V_4 = V_1$, $\longrightarrow \frac{V_3}{T_4} = \frac{V_2}{T_1}$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad or \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

The Otto cycle efficiency becomes

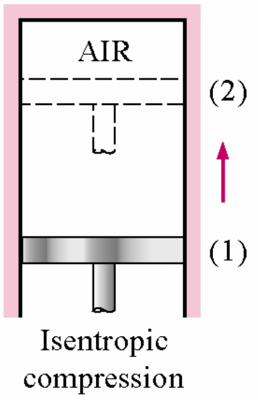
$$\eta_{th,\,Otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

$\longrightarrow$

$$\eta_{th,\,Otto} = 1 - \frac{T_1}{T_2}$$

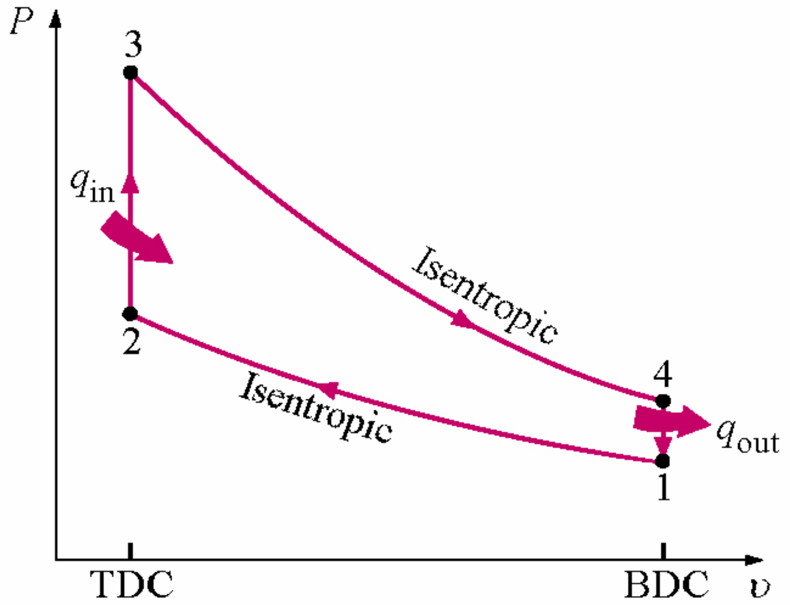
Since process 1-2 is isentropic,



$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \left(\frac{1}{r}\right)^{k-1}$$

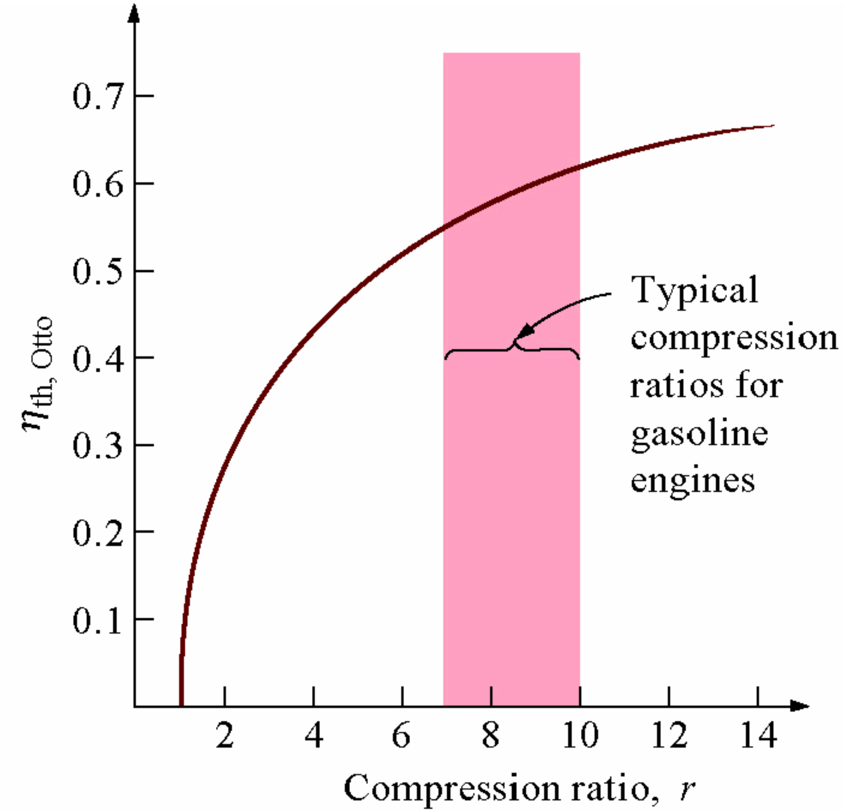
where the compression ratio is $r = V_1/V_2$ and

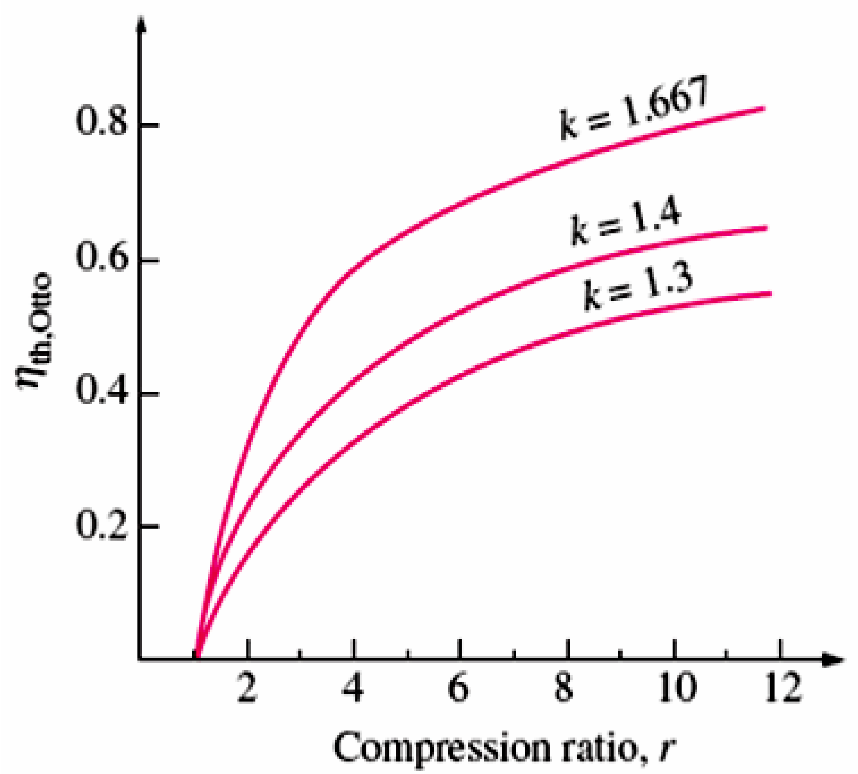


$$\eta_{th,\,Otto} = 1 - \frac{1}{r^{k-1}}$$

Thermal efficiency of the ideal Otto cycle

- Increasing the compression ratio increases the thermal efficiency.
- But there is a limit on *r* depending upon the fuel. Fuels under high temperature resulting from high compression ratios will prematurely ignite, causing knock (auto ignition).



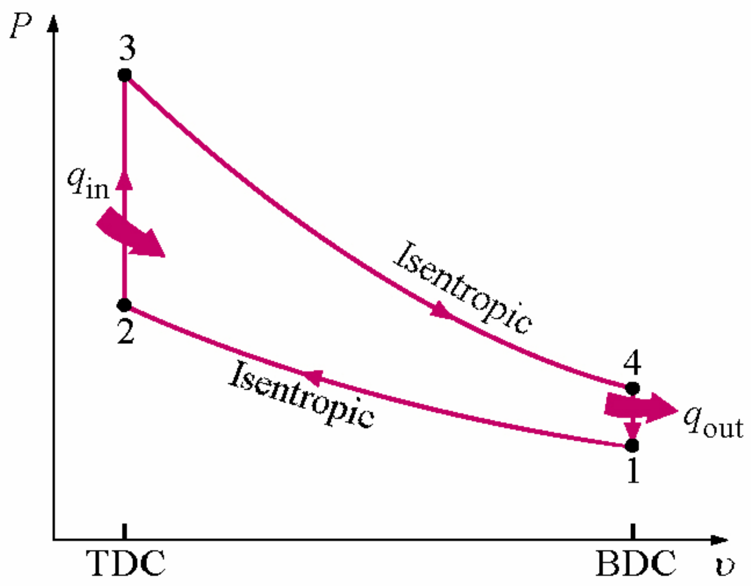


Example

An Otto cycle having a compression ratio of 9:1 uses air as the working fluid. Initially P_1 = 95 kPa, T_1 = 17°C, and V_1 = 3.8 liters. During the heat addition process, 7.5 kJ of heat are added. Determine all T's, P's, η_{th}, the back work ratio, and the mean effective pressure.

Assume constant specific heats with C_v = 0.718 kJ/kg ·K, k = 1.4. (Use the 300 K data from Table)

Process 1-2 is isentropic; therefore, recalling that $r = V_1/V_2 = 9$,

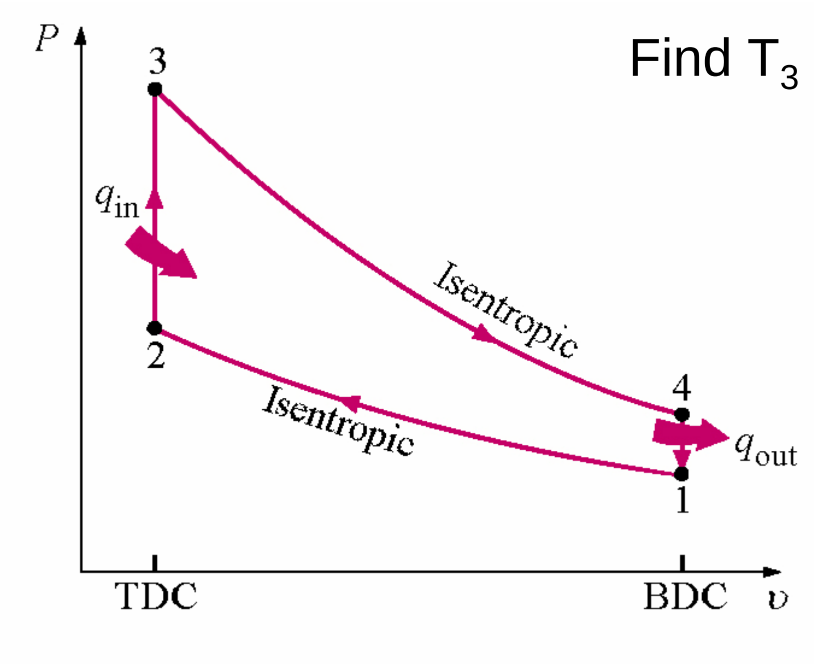


Find T_2

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = T_1(r)^{k-1}$$
$$= (17+273)K(9)^{1.4-1}$$
$$= 698.4K$$

Find P_2

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{k} = P_1(r)^{k}$$
$$= 95kPa(9)^{1.4}$$
$$= 2059\,kPa$$



Find T_3

$$Q_{in} = mC_v(T_3 - T_2) \rightarrow T_3 = T_2 + \frac{q_{in}}{C_v}$$

Let $q_{in} = Q_{in} / m$ and $m = V_1/v_1$

$$v_1 = \frac{RT_1}{P_1}$$

$$= \frac{0.287 \frac{kJ}{kg \cdot K}(290\,K)}{95\,kPa} \frac{m^3 kPa}{kJ}$$

$$= 0.875 \frac{m^3}{kg}$$

$$q_{in} = \frac{Q_{in}}{m} = Q_{in}\frac{v_1}{V_1}$$

$$= 7.5kJ \frac{0.875 \frac{m^3}{kg}}{3.8 \cdot 10^{-3} m^3}$$

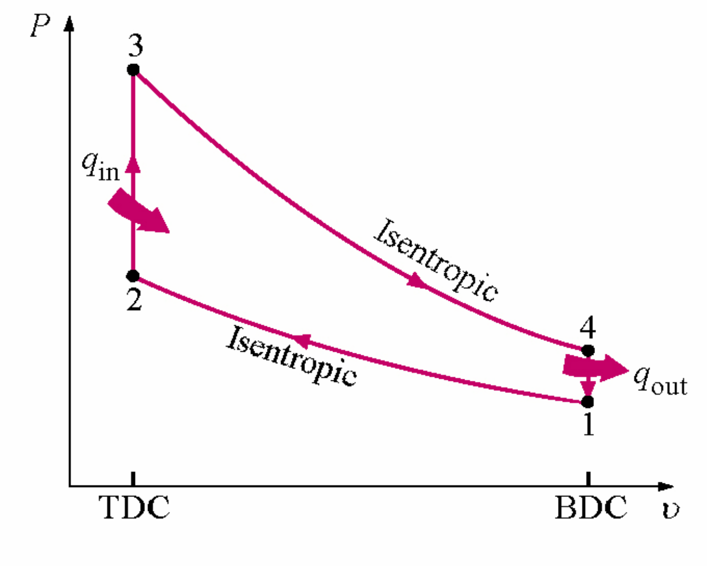
$$= 1727 \frac{kJ}{kg}$$

$$T_3 = T_2 + \frac{q_{in}}{C_v}$$

$$= 698.4\,K + \frac{1727 \frac{kJ}{kg}}{0.718 \frac{kJ}{kg \cdot K}}$$

$$= 3103.7\,K$$

Using the combined gas law ($V_3 = V_2$)



- Find P_3

$$P_3 = P_2 \frac{T_3}{T_2} = 9.15\, MPa$$

Process 3-4 is isentropic; therefore,

- Find T_4

$$T_4 = T_3 \left(\frac{V_3}{V_4} \right)^{k-1} = T_3 \left(\frac{1}{r} \right)^{k-1} = (3103.7) K \left(\frac{1}{9} \right)^{1.4-1}$$

$$= 1288.8\, K$$

- Find P_4

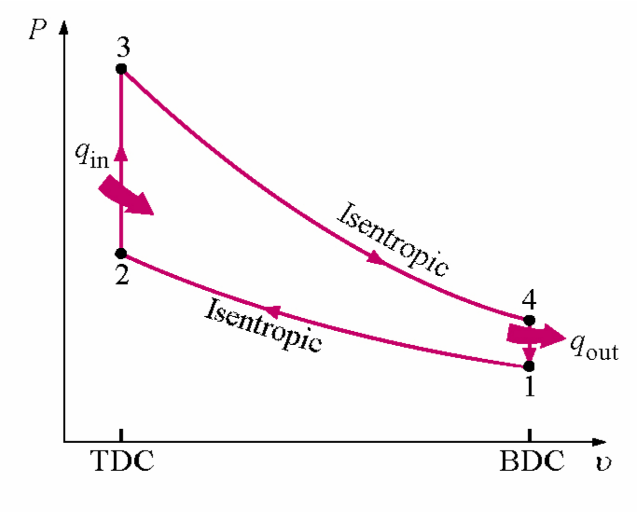
$$P_4 = P_3 \left(\frac{V_3}{V_4} \right)^{k} = P_3 \left(\frac{1}{r} \right)^{k}$$

$$= 9.15\, MPa \left(\frac{1}{9} \right)^{1.4}$$

$$= 422\, kPa$$

In order to find η_{th} , we must know net work first $\qquad w_{net} = q_{net} = q_{in} - q_{out}$

Process 4-1 is constant volume. So the first law for the closed system gives, on a mass basis,



Find q_{out}

$$Q_{out} = mC_v(T_4 - T_1)$$

$$q_{out} = \frac{Q_{out}}{m} = C_v(T_4 - T_1)$$

$$= 0.718 \frac{kJ}{kg \cdot K}(1288.8 - 290)K$$

$$= 717.1 \frac{kJ}{kg}$$

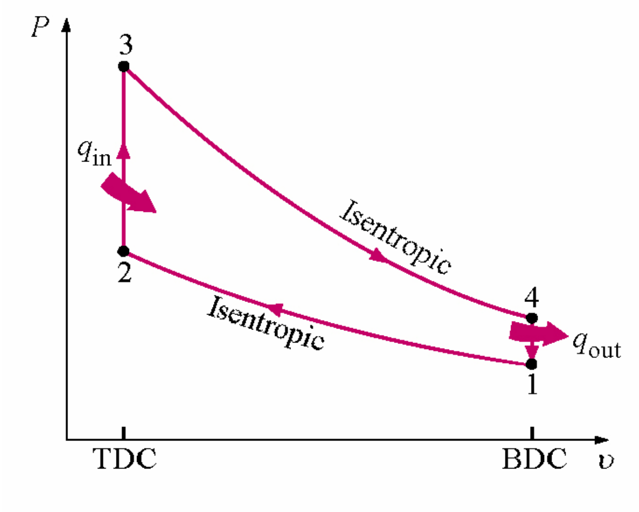
The first law applied to the cycle gives (Recall $\Delta u_{cycle} = 0$)

$$w_{net} = q_{net} = q_{in} - q_{out}$$

$$= (1727 - 717.4)\frac{kJ}{kg}$$

$$= 1009.6 \frac{kJ}{kg}$$

The thermal efficiency is



$$\eta_{th,\,Otto} = \frac{w_{net}}{q_{in}} = \frac{1009.6\dfrac{kJ}{kg}}{1727\dfrac{kJ}{kg}}$$

$$= 0.585 \;\; or \;\; 58.5\%$$

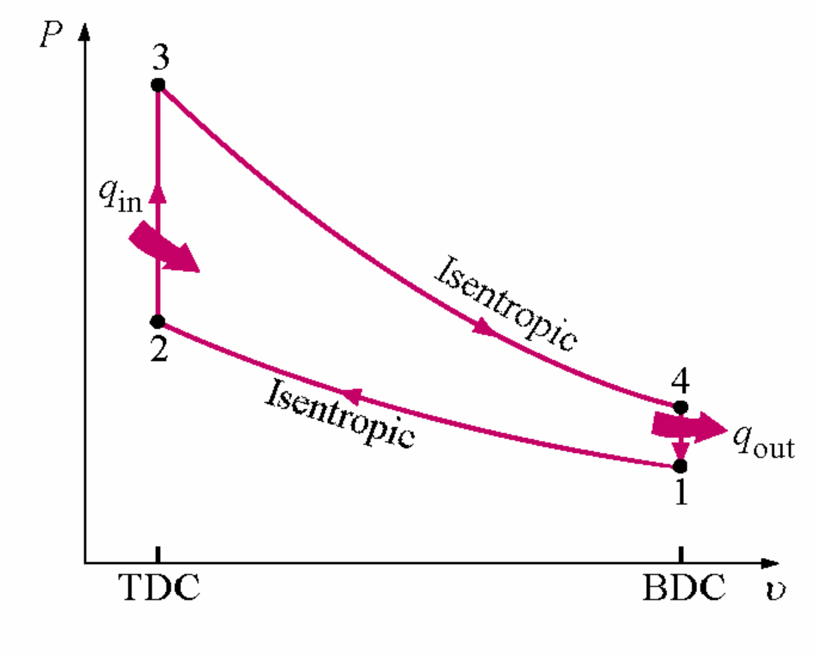
The mean effective pressure is

$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}} = \frac{w_{net}}{v_{\max} - v_{\min}} = \frac{w_{net}}{v_1 - v_2}$$

$$= \frac{w_{net}}{v_1(1 - v_2/v_1)} = \frac{w_{net}}{v_1(1 - 1/r)}$$

$$= \frac{1009.6\dfrac{kJ}{kg}}{0.875\dfrac{m^3}{kg}(1 - \dfrac{1}{9})} \frac{m^3 kPa}{kJ} = 1298 \; kPa$$

The back work ratio is

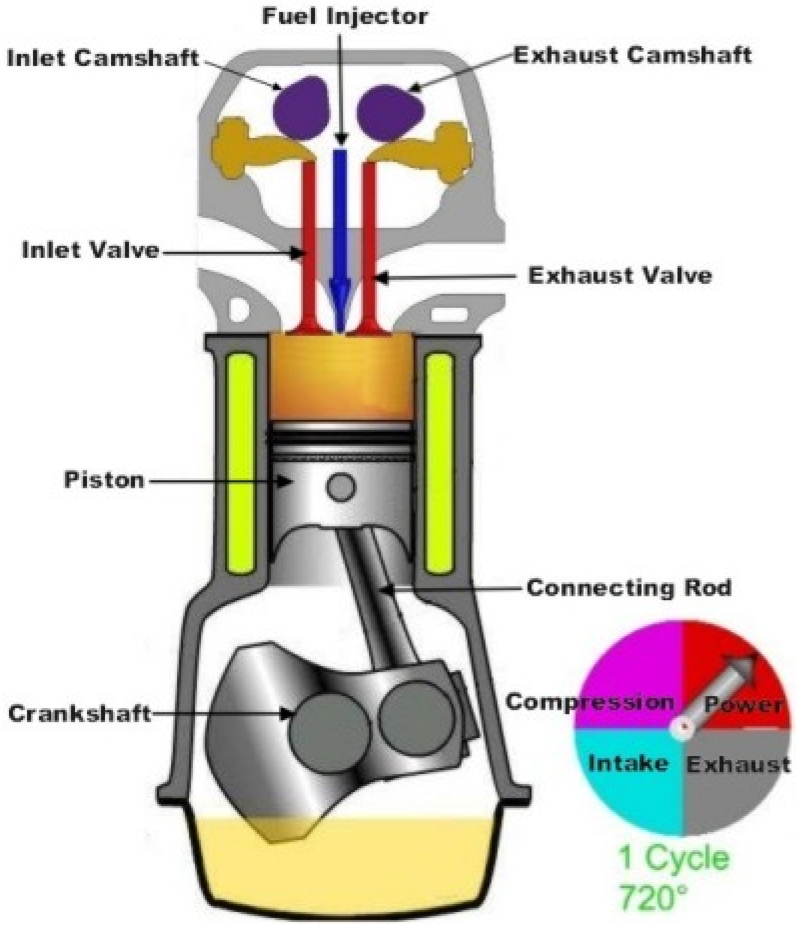


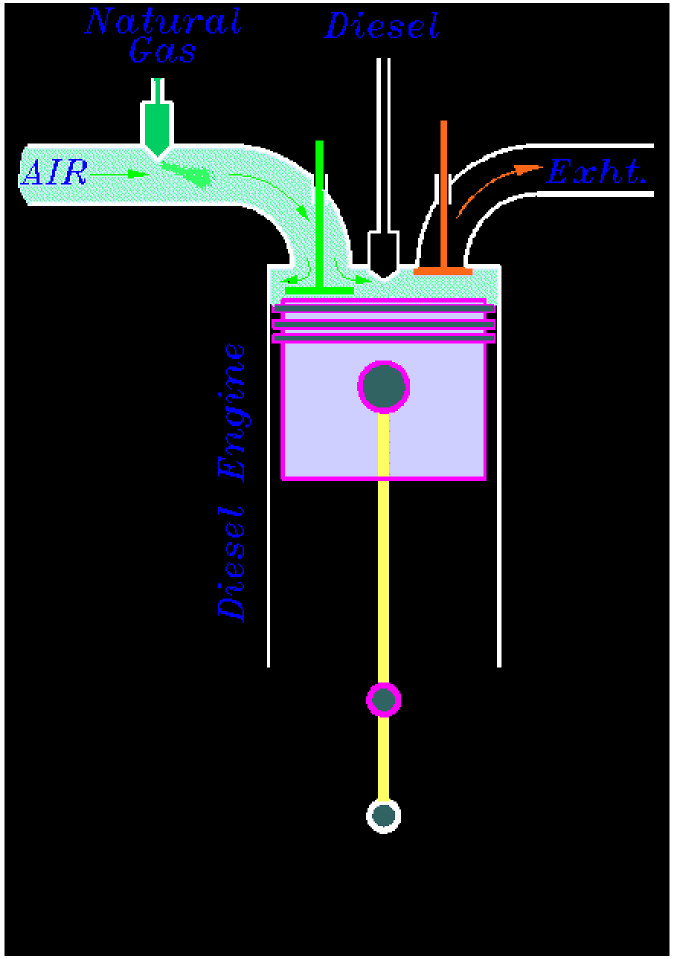
$$BWR = \frac{w_{comp}}{w_{\exp}}$$

$$= \frac{\Delta u_{12}}{-\Delta u_{34}} = \frac{C_v(T_2 - T_1)}{C_v(T_3 - T_4)} = \frac{(T_2 - T_1)}{(T_3 - T_4)}$$

$$= 0.225 \;\; or \;\; 22.5\%$$

Diesel Cycle



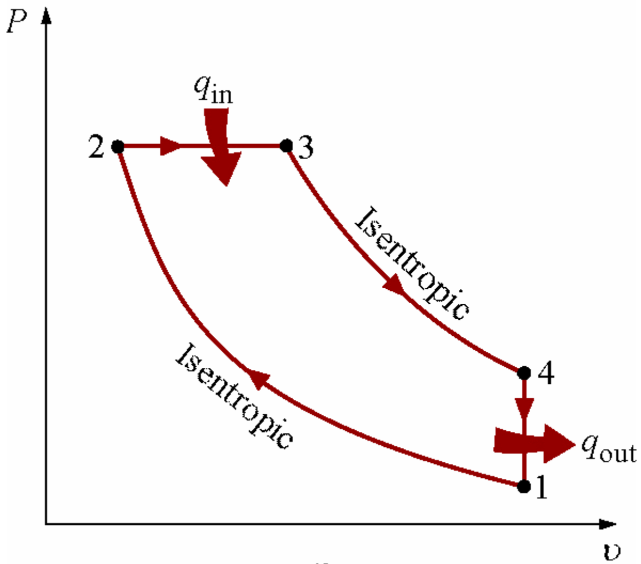


Air-Standard Diesel Cycle

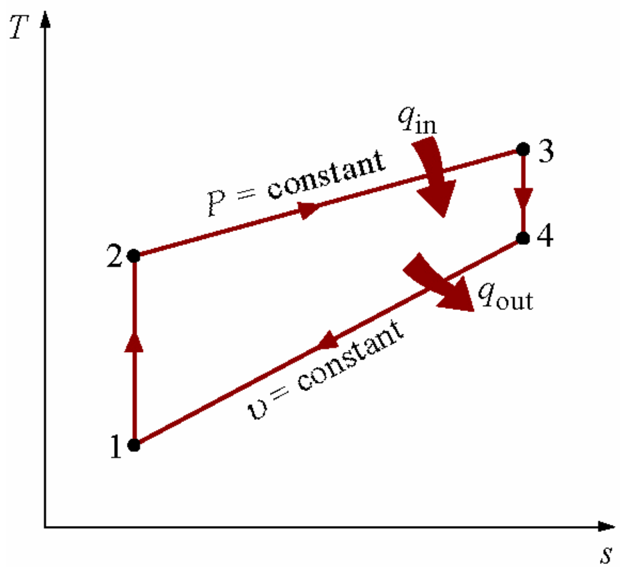
The air-standard Diesel cycle is the ideal cycle that approximates the Diesel combustion engine

Process	Description
1-2	Isentropic compression
2-3	Constant pressure heat addition
3-4	Isentropic expansion
4-1	Constant volume heat rejection

The *P-v* and *T-s* diagrams are

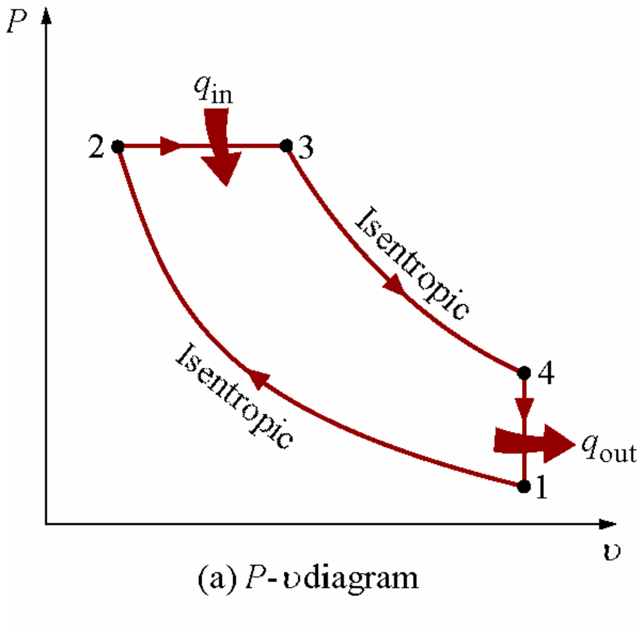


(a) P-υ diagram



(b) T-s diagram

Thermal efficiency of the Diesel cycle



$$\eta_{th,\ Diesel} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Find Q_{in}

Apply the first law closed system to process 2-3, P = constant.

$$E_{in} - E_{out} = \Delta E$$

$$Q_{net,23} - W_{net,23} = \Delta U$$

$$W_{net,23} = W_{other,23} + W_{b,23} = 0 + \int_2^3 PdV$$

$$= P_2\left(V_3 - V_2\right)$$

Thus, for constant specific heats

$$Q_{net,\ 23} = \Delta U_{23} + P_2(V_3 - V_2)$$

$$Q_{net,\ 23} = Q_{in} = mC_v(T_3 - T_2) + mR(T_3 - T_2)$$

$$Q_{in} = mC_p(T_3 - T_2)$$

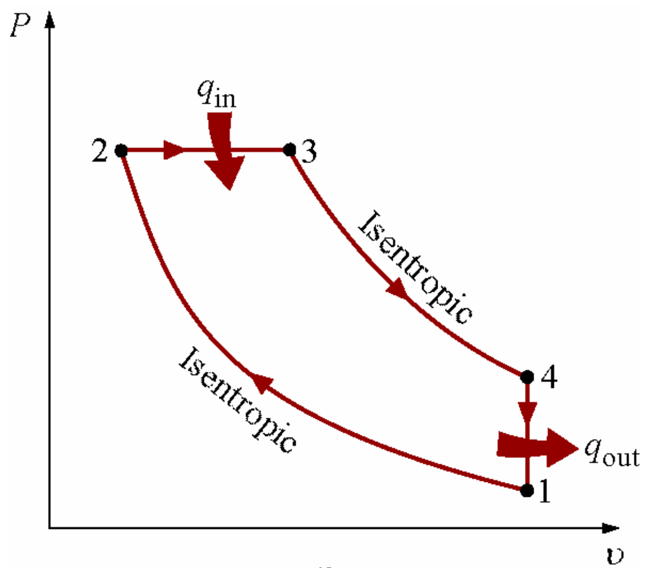
Find Q_{out}

Apply the first law closed system to process 4-1, V = constant

$$E_{in} - E_{out} = \Delta E$$

$$Q_{net,41} - W_{net,41} = \Delta U_{41}$$

$$W_{net,41} = W_{other,41} + W_{b,41} = 0 + \int_4^1 PdV$$



(a) P-υ diagram

Thus, for constant specific heats

$$Q_{net,\,41} = \Delta U_{41}$$

$$Q_{net,\,41} = -Q_{out} = mC_v(T_1 - T_4)$$

$$Q_{out} = -mC_v(T_1 - T_4) = mC_v(T_4 - T_1)$$

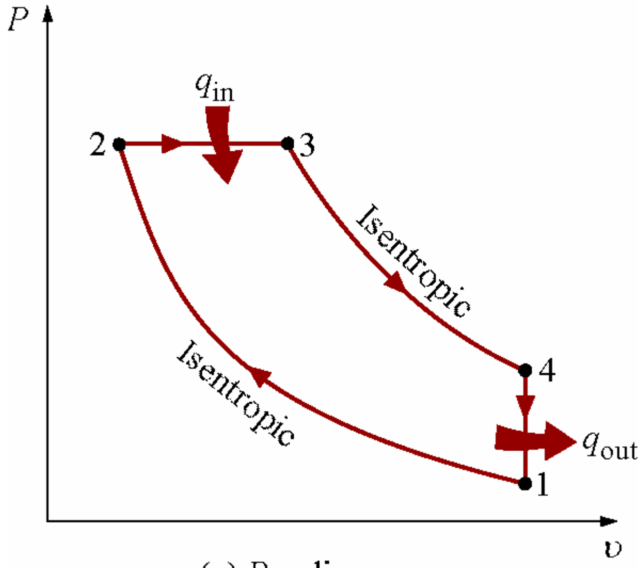
The thermal efficiency becomes

$$\eta_{th,\,Diesel} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{mC_v(T_4 - T_1)}{mC_p(T_3 - T_2)}$$

$$\eta_{th,\,Diesel} = 1 - \frac{C_v (T_4 - T_1)}{C_p (T_3 - T_2)}$$

$$= 1 - \frac{1}{k} \frac{T_1 (T_4 / T_1 - 1)}{T_2 (T_3 / T_2 - 1)}$$



(a) P-υdiagram

$$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2} \quad \text{where } P_3 = P_2$$

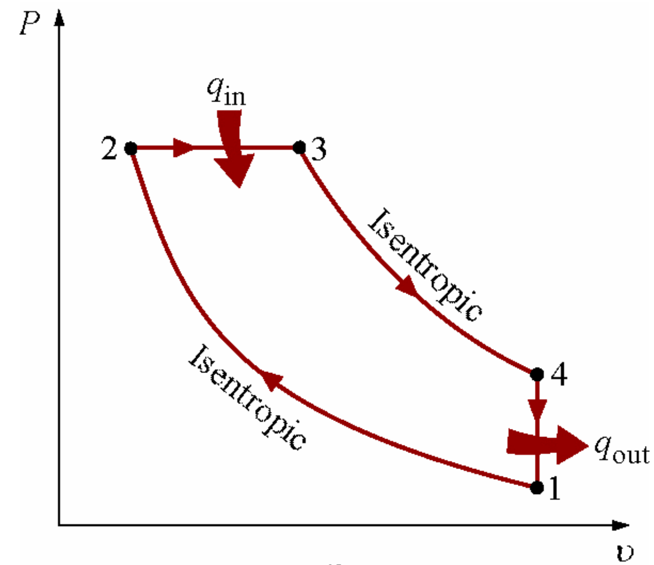
$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c$$

where $\boldsymbol{r_c}$ **is called the cutoff ratio**, defined as V_3 / V_2, and **is a measure of the duration of the heat addition at constant pressure**. Since the fuel is injected directly into the cylinder, the cutoff ratio can be related to the number of degrees that the crank rotated during the fuel injection into the cylinder.

$$\eta_{th,\,Diesel} = 1 - \frac{1}{k}\frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$= 1 - \frac{1}{k}\frac{T_1}{T_2}\frac{r_c^k - 1}{(r_c - 1)}$$

$$= 1 - \frac{1}{r^{k-1}}\frac{r_c^k - 1}{k(r_c - 1)}$$

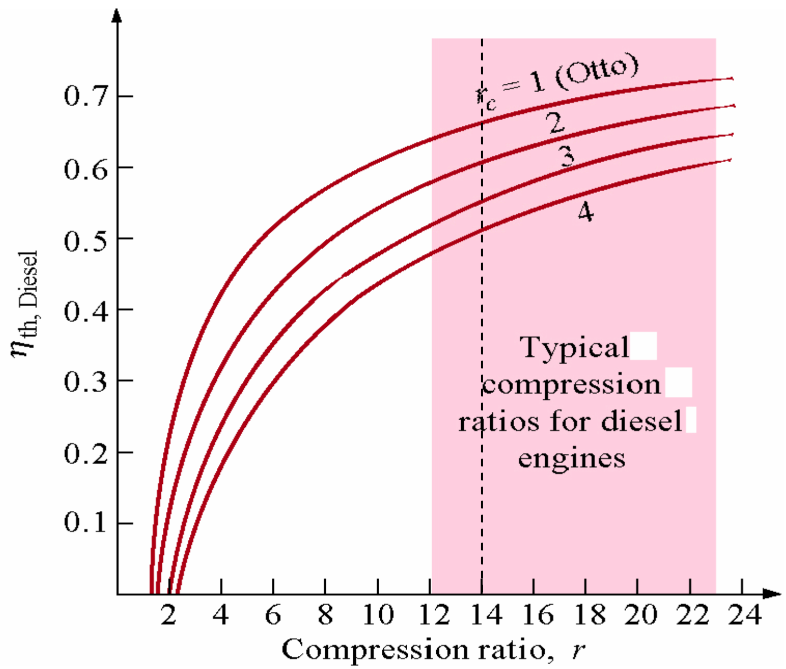


(a) P-υ diagram

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c$$

When $r_c > 1$ for a fixed r, $\eta_{th,Diesel} < \eta_{th,Otto}$

But, since $r_{Diesel} > r_{Otto}$, $\eta_{th,Diesel} > \eta_{th,Otto}$.



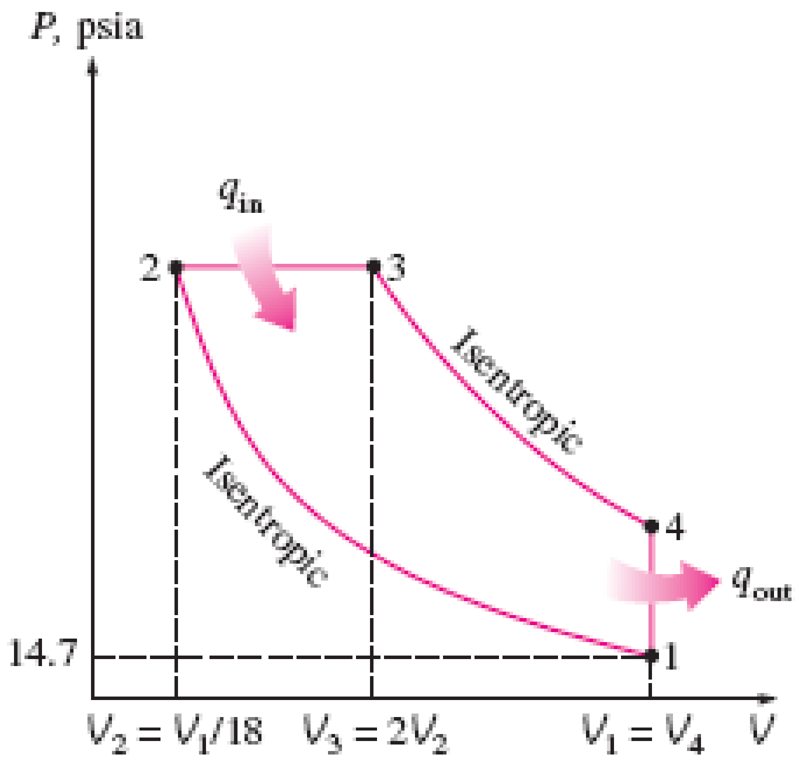
Example

An ideal Diesel cycle with air as the working fluid has a compression ratio of 18 and a cutoff of 2. At the beginning of the compression process, the working fluid is at 14.7 psia, 80 ^{O}F,and 117 in^3. Utilizing the cold-air-standard assumptions, determine

(a) the temperature and pressure of air at the end of each process,

(b) the net work output and the thermal efficiency,

(c) the mean effective pressure



Properties

R = 0.3704 psia. ft^3/lbm.R

At room temp., c_p = 0.204 Btu/lbm.R

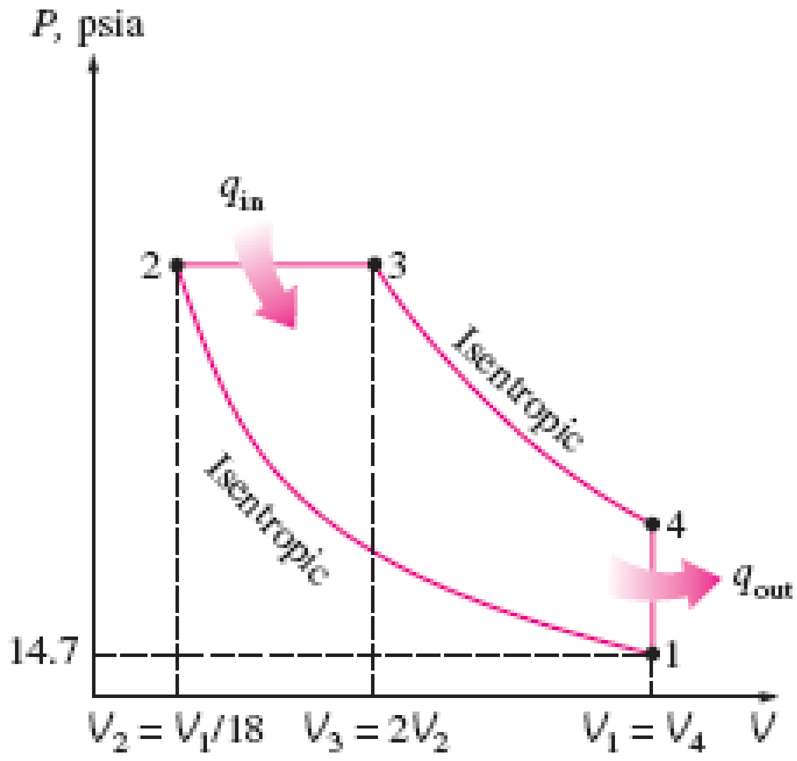
c_v = 0.171 Btu/lbm.R

k = 1.4

From compression ratio, r = 18

$$r = \frac{V_{\max}}{V_{\min}} = \frac{V_1}{V_2}$$

$$V_2 = \frac{V_1}{r} = \frac{117\ \text{in}^3}{18} = 6.5\ \text{in}^3$$



From cutoff ratio, r_c= 2

$$r_c = \frac{V_3}{V_2}$$

$$V_3 = (2)\left(6.5\ \text{in}^3\right) = 13\ \text{in}^3$$

$$V_4 = V_1 = 117\ \text{in}^3$$

T_2 and P_2

Process 1-2 (isentropic compression of an ideal gas, constant specific heat)

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = (540\ \text{R})(18)^{1.4-1} = 1716\ \text{R}$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{k} = (14.7\ psia)(18)^{1.4} = 841\ psia$$

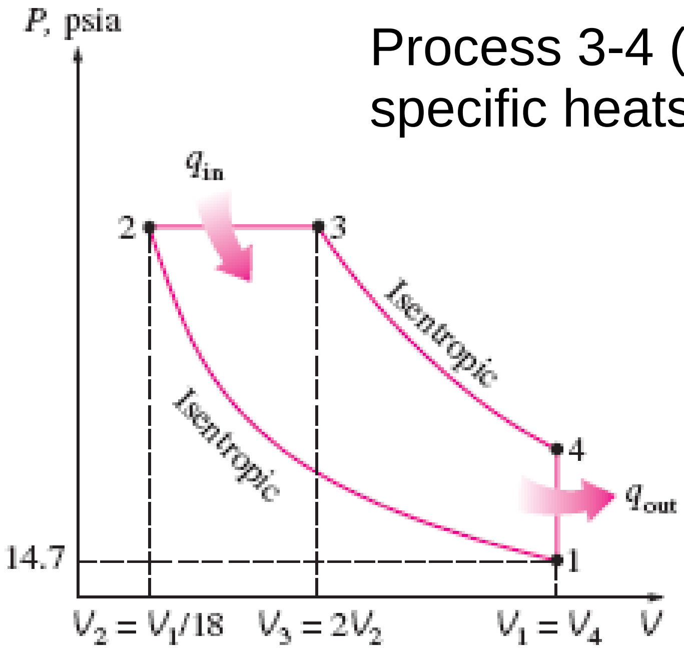
T_3 and P_3

Process 2-3 (constant pressure heat addition to an ideal gas)

$$P_3 = P_2 = 841\ psia$$

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \rightarrow \quad T_3 = T_2\left(\frac{V_3}{V_2}\right) = (1716\ \text{R})(2) = 3432\ \text{R}$$

T_4 and P_4



Process 3-4 (isentropic expansion of an ideal gas, constant specific heats)

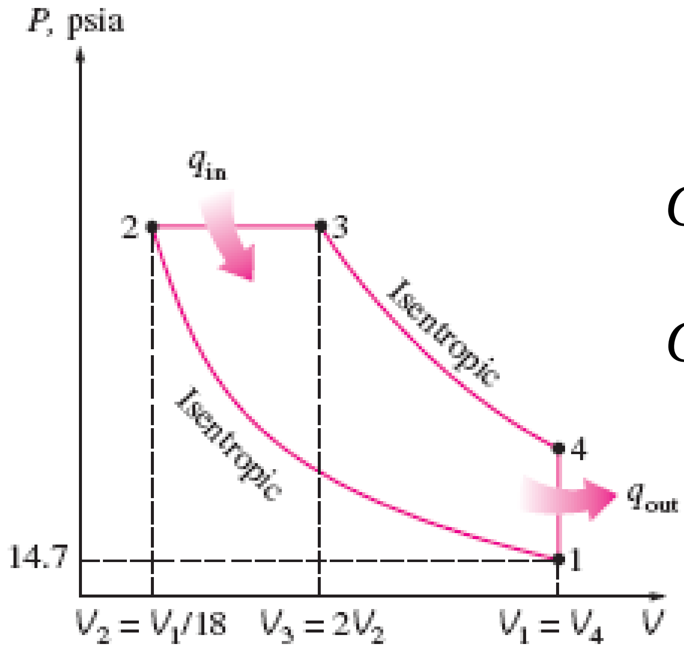
$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{k-1} = (3432\ \text{R})\left(\frac{13}{117}\right)^{1.4-1} = 1425\ \text{R}$$

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{k} = (841\ psia)\left(\frac{13}{117}\right)^{1.4} = 38.8\ psia$$

(b) Net work $W_{net} = Q_{in} - Q_{out}$

$$Q_{in} = m\left(h_3 - h_2\right) = mc_p\left(T_3 - T_2\right)$$

$$m = \frac{P_1V_1}{RT_1} = \frac{(14.7\text{ psia})(117)}{\left(0.3704\text{ psia.ft}^3\text{/lbm.R}\right)(540\text{ R})}\left(\frac{1\text{ ft}^3}{1728\text{ in}^3}\right) = 0.00498\text{ lbm}$$



$$Q_{in} = (0.00498)(0.24)(3432 - 1716) = 2.051\text{ Btu}$$

$$Q_{out} = m\left(u_4 - u_1\right) = mc_v\left(T_4 - T_1\right)$$

$$Q_{out} = (0.00498)(0.171)(1425 - 540) = 0.754\text{ Btu}$$

$$W_{net} = 2.051 - 0.754 = 1.297\text{ Btu}$$

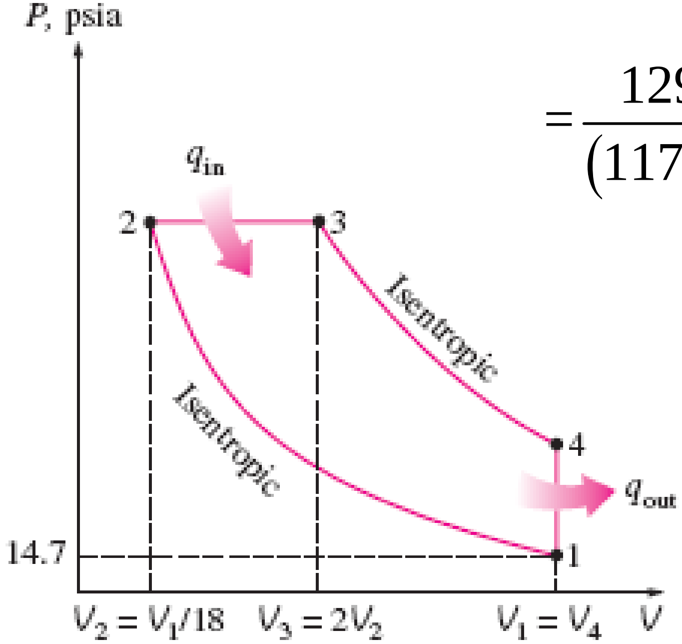
Thermal efficiency

$$\eta_{th,\,Diesel} = \frac{W_{net}}{Q_{in}} = \frac{1.297}{2.051} = 0.632$$

(c) Mean effective pressure

$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}} = \frac{W_{net}}{V_1 - V_2}$$

$$= \frac{1297 \text{ Btu}}{(117\text{-}6.5)\text{in}^3}\left(\frac{778.17 \text{ lbf.ft}}{1 \text{ Btu}}\right)\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = 110 \text{ psia}$$



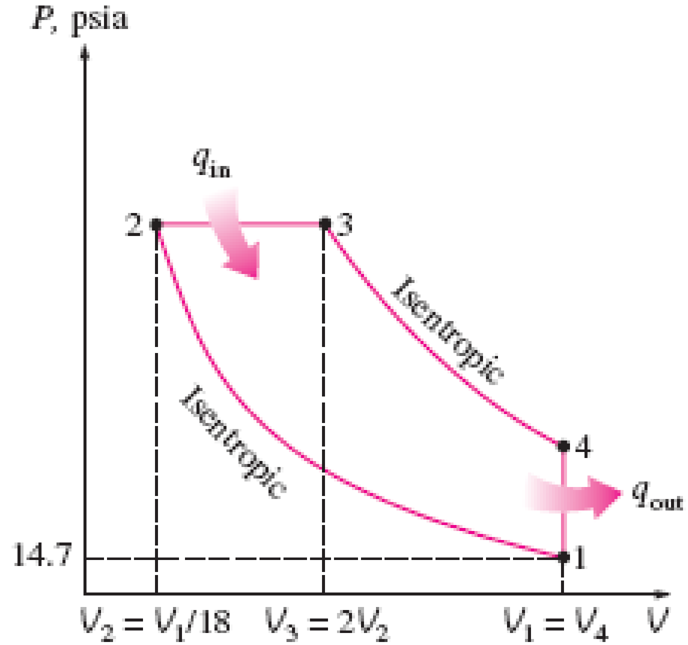
Summary of Diesel cycle

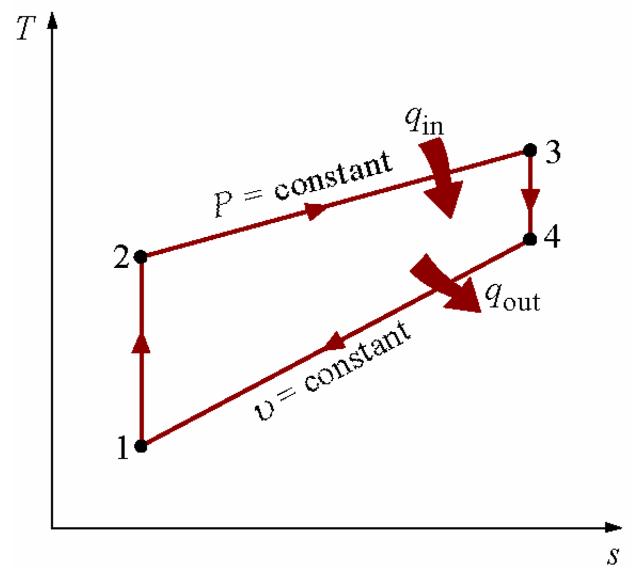
Cutoff ratio, r_c

Defined as V_3 / V_2, and is a measure of the duration of the heat addition at constant pressure.

Thermal efficiency

$$\eta_{th,Diesel} = 1 - \frac{1}{r^{k-1}} \frac{r_c^k - 1}{k(r_c - 1)}$$

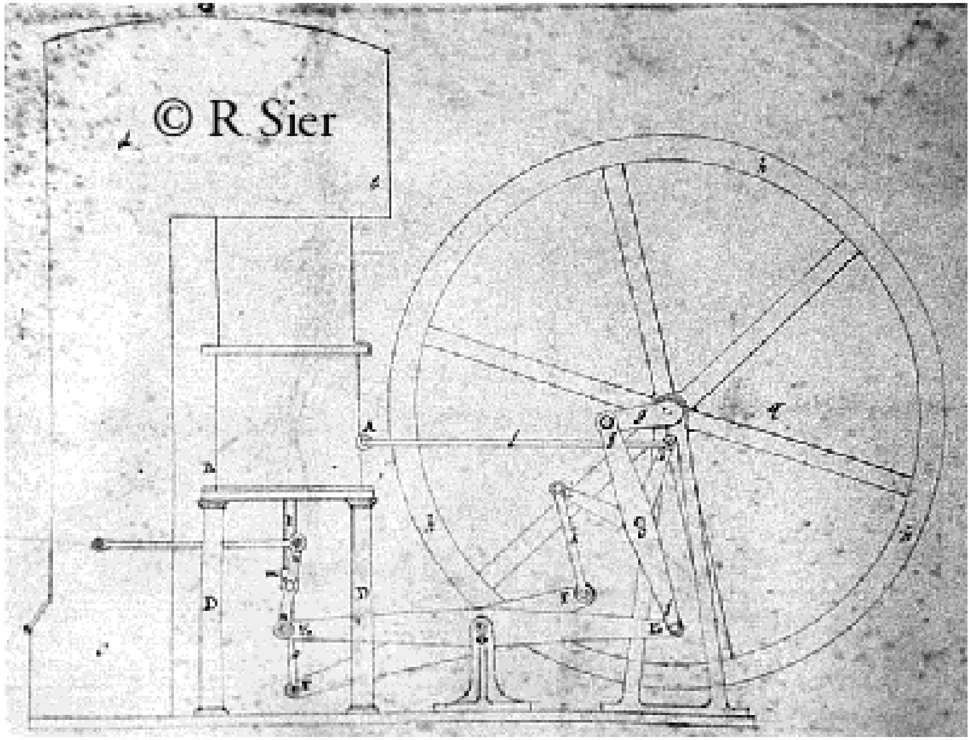


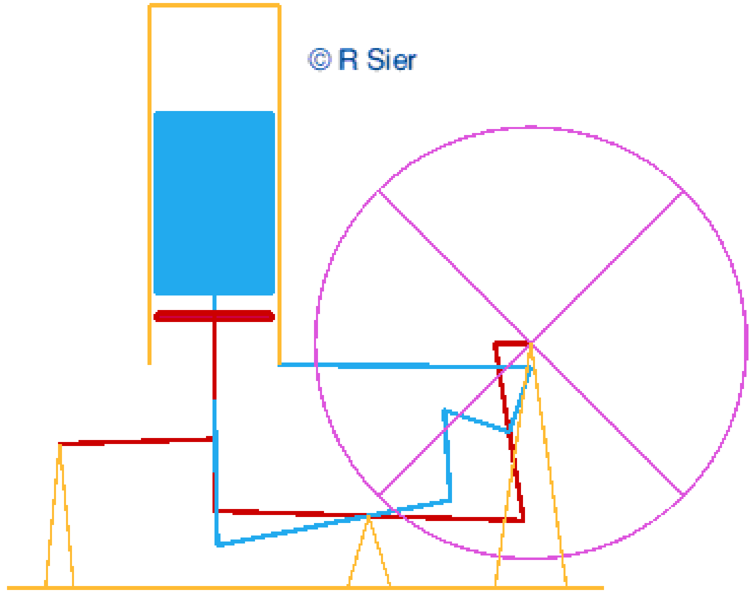


(b) *T*-*s* diagram

Stirling engine

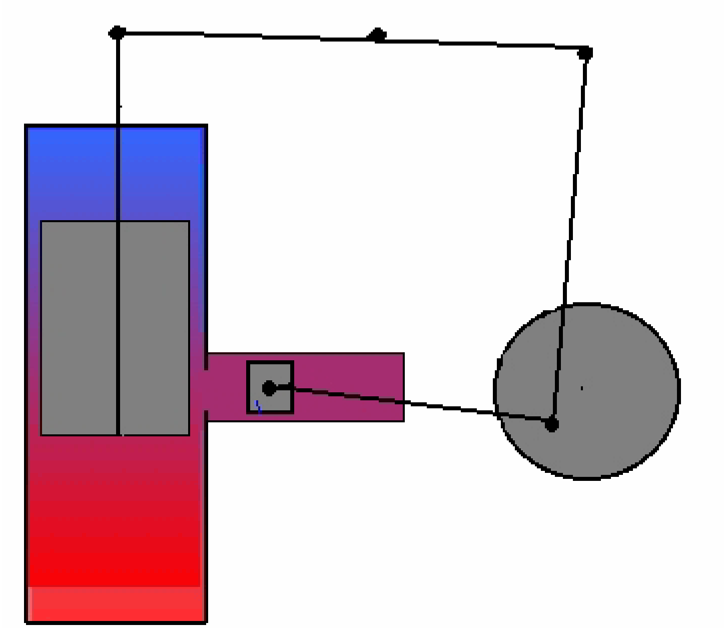
Robert Stirling patented his Heat Economiser in 1816

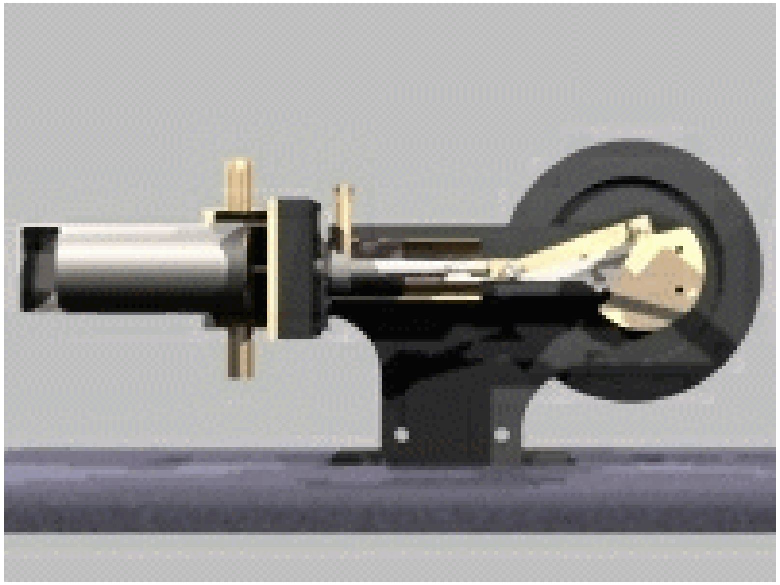




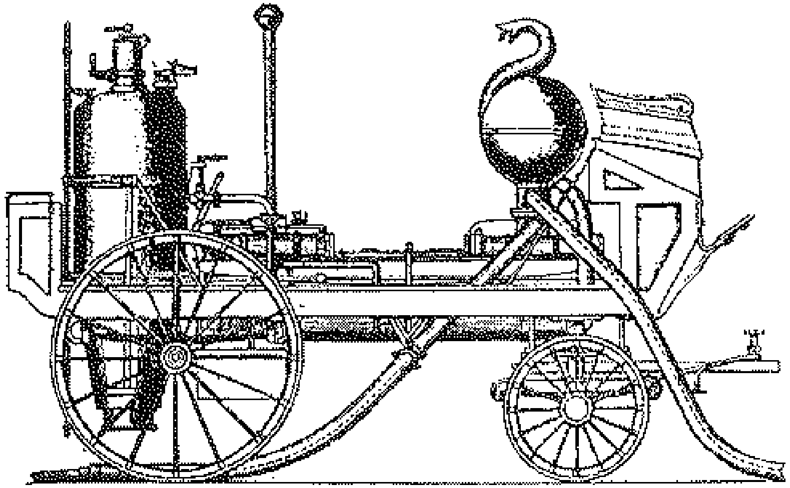
The power piston compresses the enclosed air in the cold end of the displacer cylinder. The displacer then shifts the air from cold to hot chambers. The piston is driven back, the power stroke, by the air expanding in the hot end.

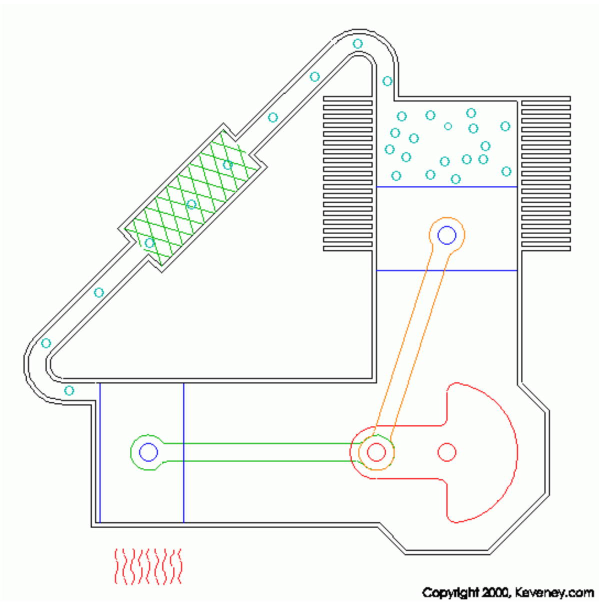
Stirling engine

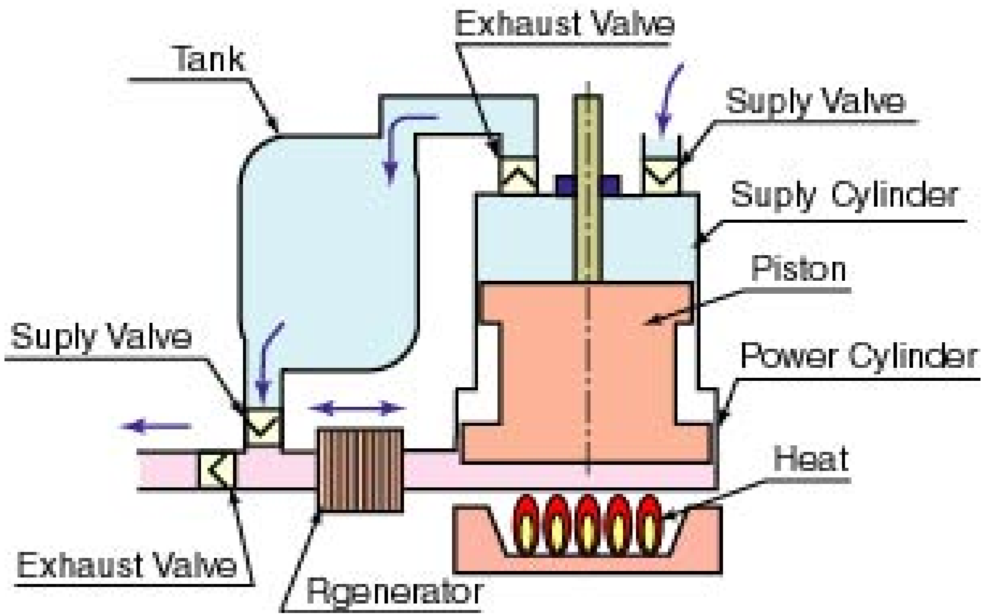




Ericsson Engine



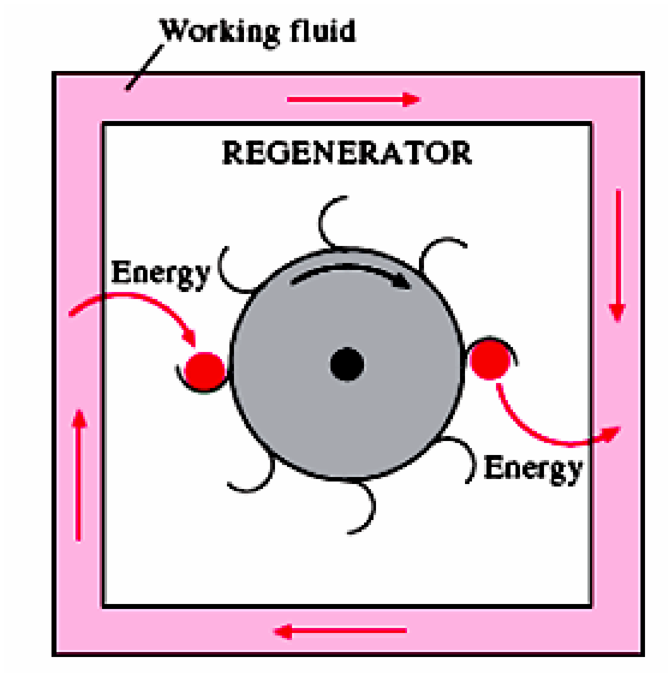




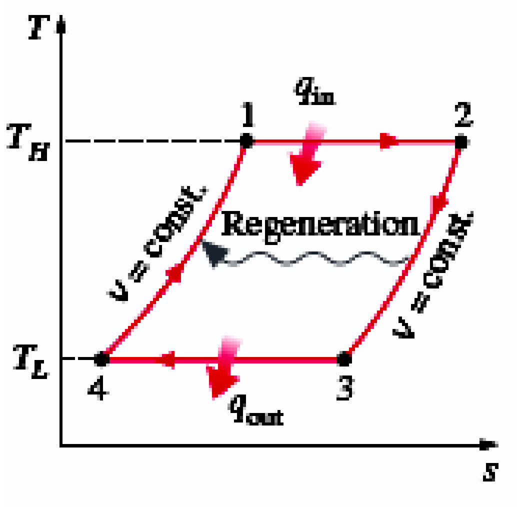
Stirling and Ericsson cycles

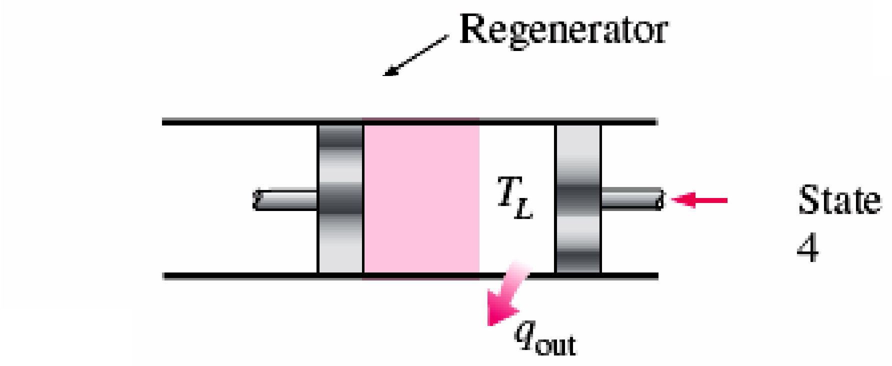
-**Involve an isothermal heat addition process at T_H and isothermal heat rejection process at T_L**

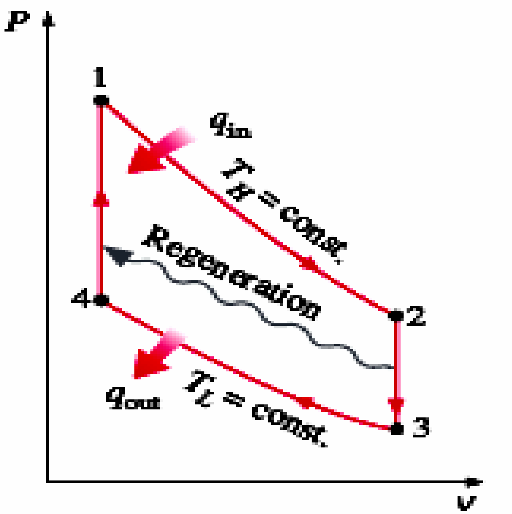
- **Both cycles utilize regeneration**
  - a process during which heat is transferred to a thermal energy storage device (called a regenerator) during one part of the cycle and is transferred back to the working back to the working fluid during the other part of the cycle.



Ideal stirling cycle

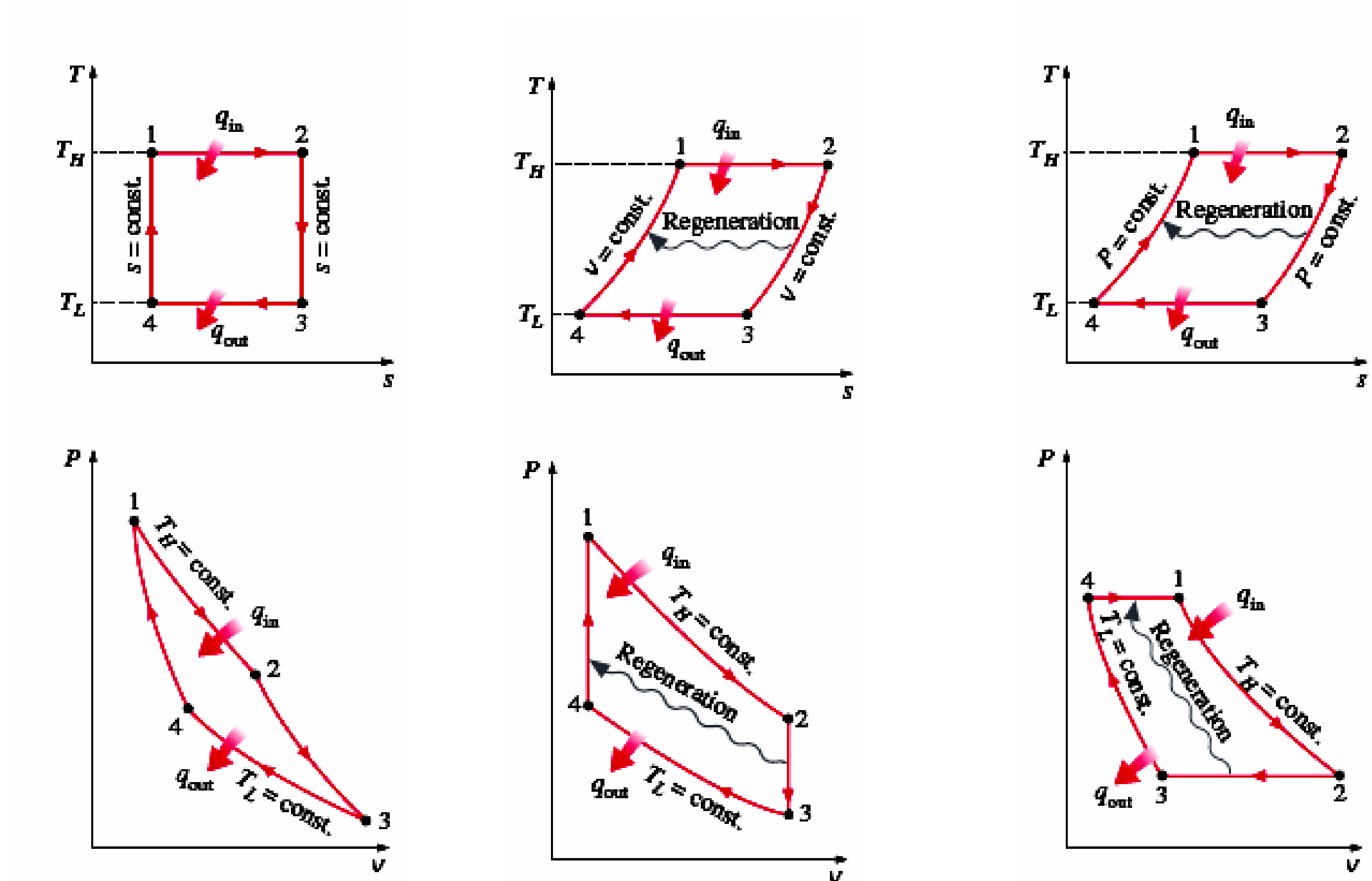






(b) Stirling cycle

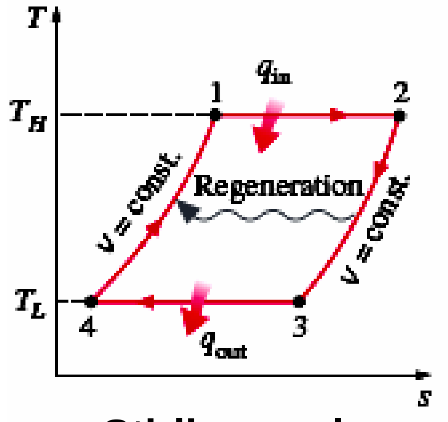
1-2	T = constant expansion (heat addition from the external source)
2-3	v = constant regeneration (internal heat transfer from the working fluid to the regenerator
3-4	T = constant compression (heat rejection to the external sink)
4-1	v = constant regeneration (internal heat transfer from the regenerator back to the working fluid)



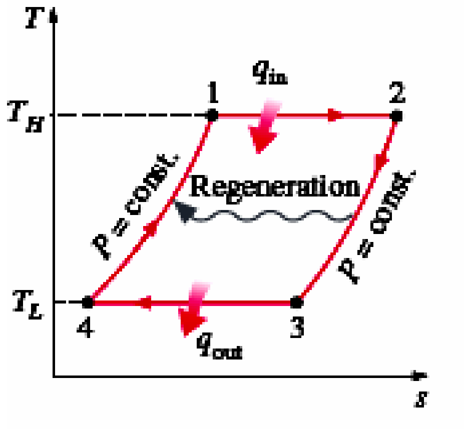
(a) Carnot cycle (b) Stirling cycle (c) Ericsson cycle

Stirling and Ericsson cycles differ from the Carnot cycle in that two isentropic processes are replaced by two constant volume regeneration processes in Stirling cycle and by two constant pressure regeneration processes in Ericsson cycle.

Thermal efficiency of Stirling and Ericsson cycles



Stirling cycle



Ericsson cycle

Process 1-2 and 3-4

$$q = T\Delta s$$

Entropy change of an ideal gas during an isothermal process

$$\Delta s = c_p \ln \frac{T_e}{T_i} - R\ln\frac{P_e}{P_i} = -R\ln\frac{P_e}{P_i}$$

$$q_{in} = T_H\left(s_2 - s_1\right) = T_H\left(-R\ln\frac{P_2}{P_1}\right) = RT_H \ln\frac{P_1}{P_2}$$

$$q_{out} = T_L\left(s_4 - s_3\right) = -T_L\left(-R\ln\frac{P_4}{P_3}\right) = RT_L \ln\frac{P_4}{P_3}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{RT_L \ln\left(P_4/P_3\right)}{RT_H \ln\left(P_1/P_2\right)} = 1 - \frac{T_L}{T_H}$$

$$\eta_{th,Stirling} = \eta_{th,Ericsson} = \eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$

Merit and demerit of Stirling and Ericsson engines

Demerit

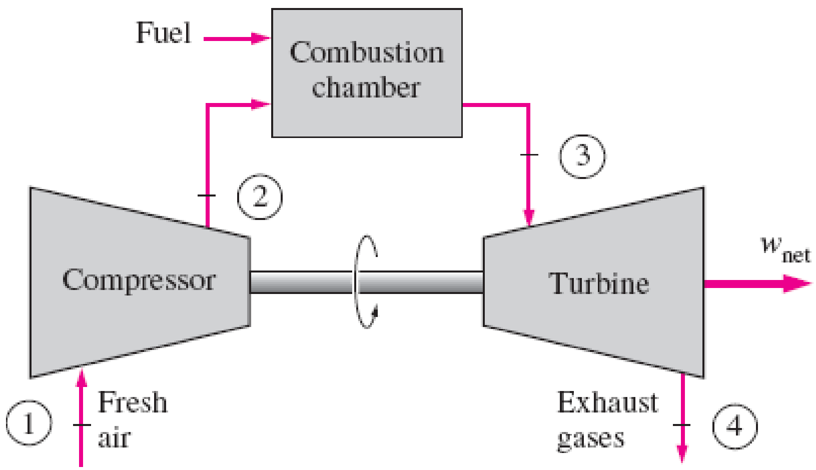
- ❑ **Difficult to achieve in practice**
  - They involve heat transfer through a differential temperature difference in all components including regenerator.
  - Need large surface for allowing heat transfer or need long time for the process.
- ❑ **Pressure losses in the generator are considerable.**

Merit

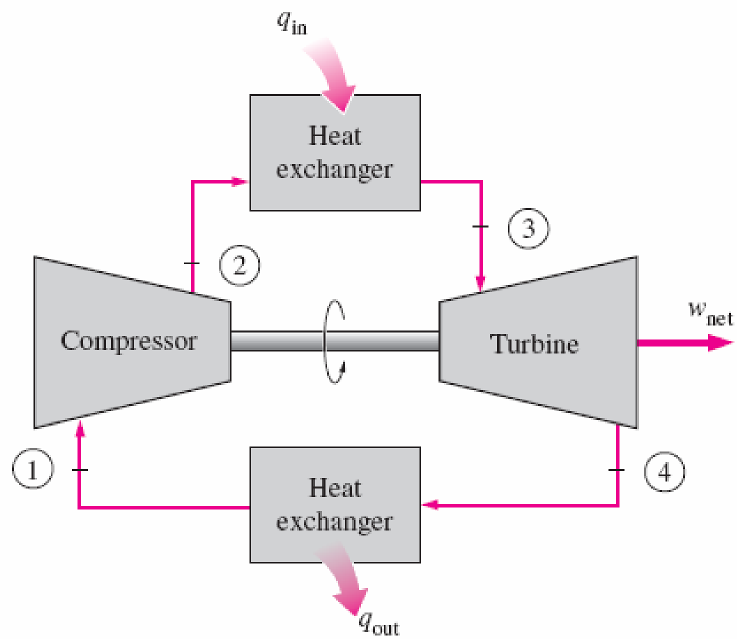
- ❑ **Due to external combustion in both cycles**
  - A variety of fuel can be used.
  - There are more time for combustion, resulting in more complete combustion (less pollution)
- ❑ **Because these cycles operate on closed cycles**,
  - A working fluid (e.g.,H and He) that has most desirable characteristics (stable, chemically inert, high thermal conductivity) can be used.

Brayton Cycle

- The Brayton cycle was first proposed by George Brayton around 1870.
- **The Brayton cycle is the air-standard ideal cycle approximation for the gas turbine engine.**
- This cycle differs from the Otto and Diesel cycles
  - Processes making the cycle occur in open systems or control volumes.



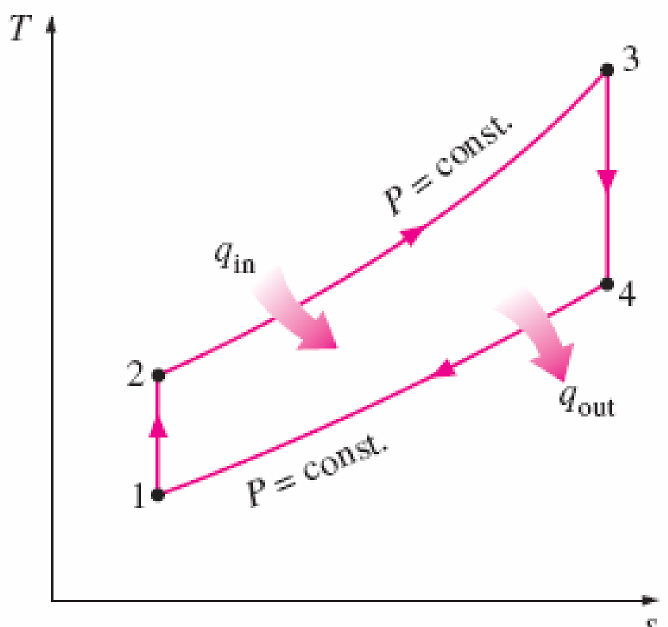
A open cycle gas turbine engine



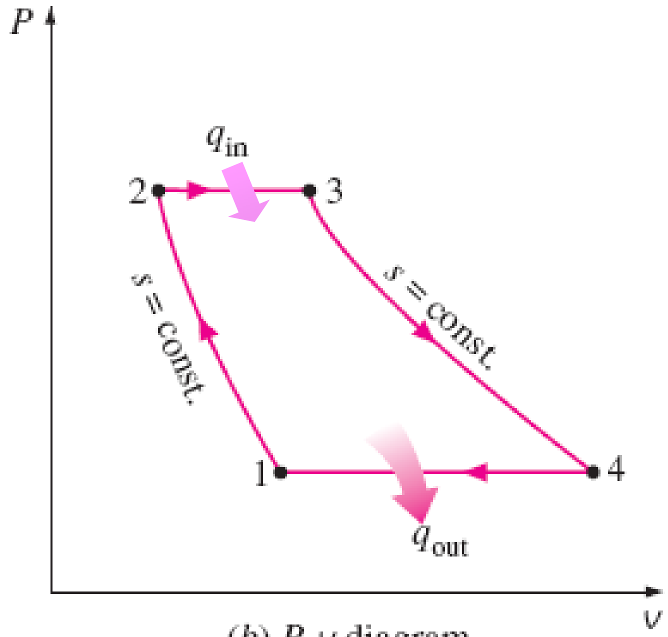
A closed cycle gas turbine engine with air standard assumptions

Brayton Cycle

1-2 Isentropic compression (in a compressor)
2-3 Constant pressure heat addition
3-4 Isentropic expansion (in a turbine)
4-1 Constant pressure heat rejection

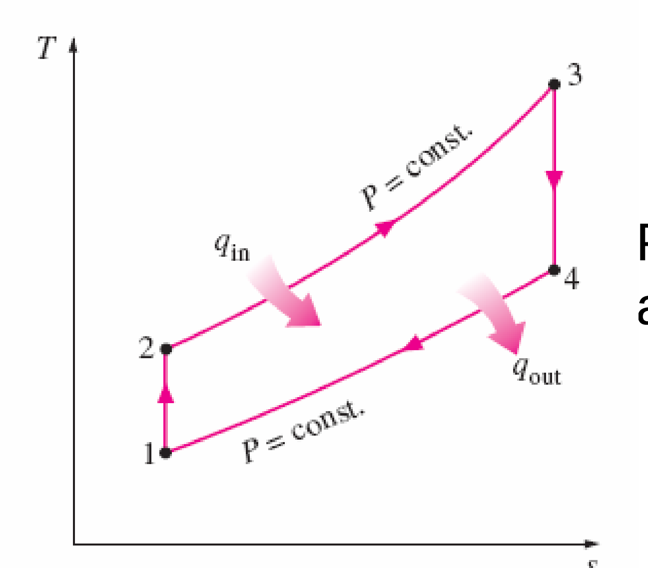


(*a*) *T*-*s* diagram



(*b*) *P*-*v* diagram

Thermal efficiency of the Brayton cycle



(a) T-s diagram

$$\eta_{th,\ Brayton} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Process 2-3 for *P* = constant (no work), steady-flow, and neglect changes in kinetic and potential energies.

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

The conservation of mass gives

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

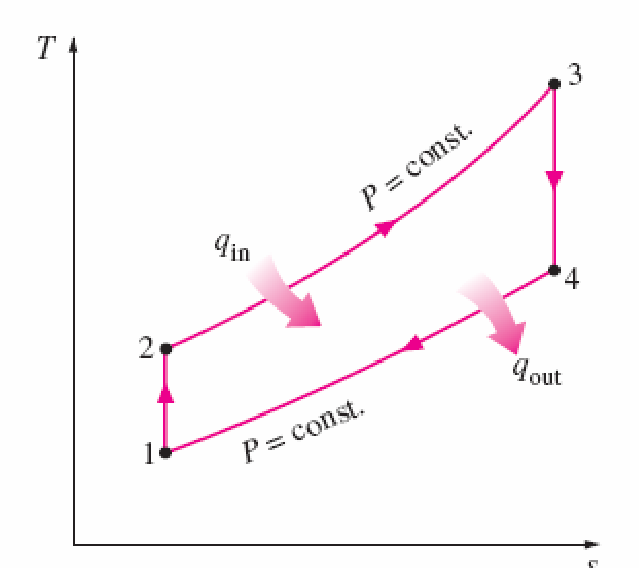
For constant specific heats, the heat added per unit mass flow is

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2)$$

$$\dot{Q}_{in} = \dot{m} C_p (T_3 - T_2)$$

$$\longrightarrow \quad q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = C_p (T_3 - T_2)$$

Process 4-1; constant specific heats



(a) T-s diagram

$$\dot{Q}_{out} = \dot{m}(h_4 - h_1)$$

$$\dot{Q}_{out} = \dot{m}C_p(T_4 - T_1)$$

$$q_{out} = \frac{\dot{Q}_{out}}{\dot{m}} = C_p(T_4 - T_1)$$

The thermal efficiency becomes

$$\eta_{th,\ Brayton} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

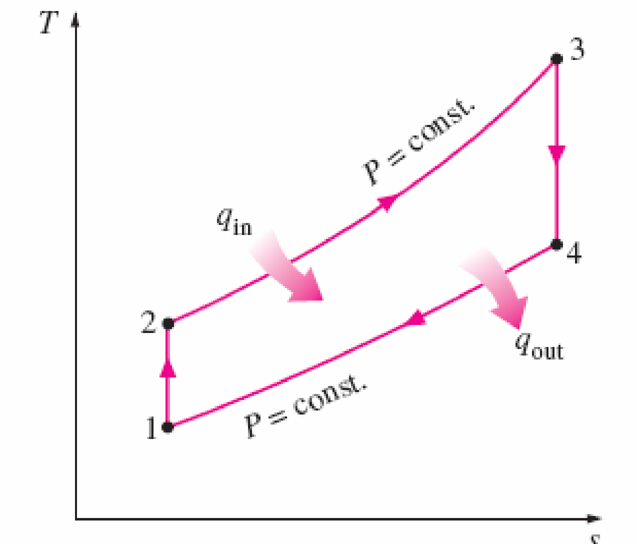
$$= 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$\eta_{th,\ Brayton} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

Processes 1-2 and 3-4 are isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \quad and \quad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k}$$



(a) T-s diagram

Since $P_3 = P_2$ and $P_4 = P_1$,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

The Brayton cycle efficiency becomes

$$\eta_{th,\ Brayton} = 1 - \frac{T_1}{T_2}$$

Process 1-2 is isentropic,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = r_p^{(k-1)/k}$$

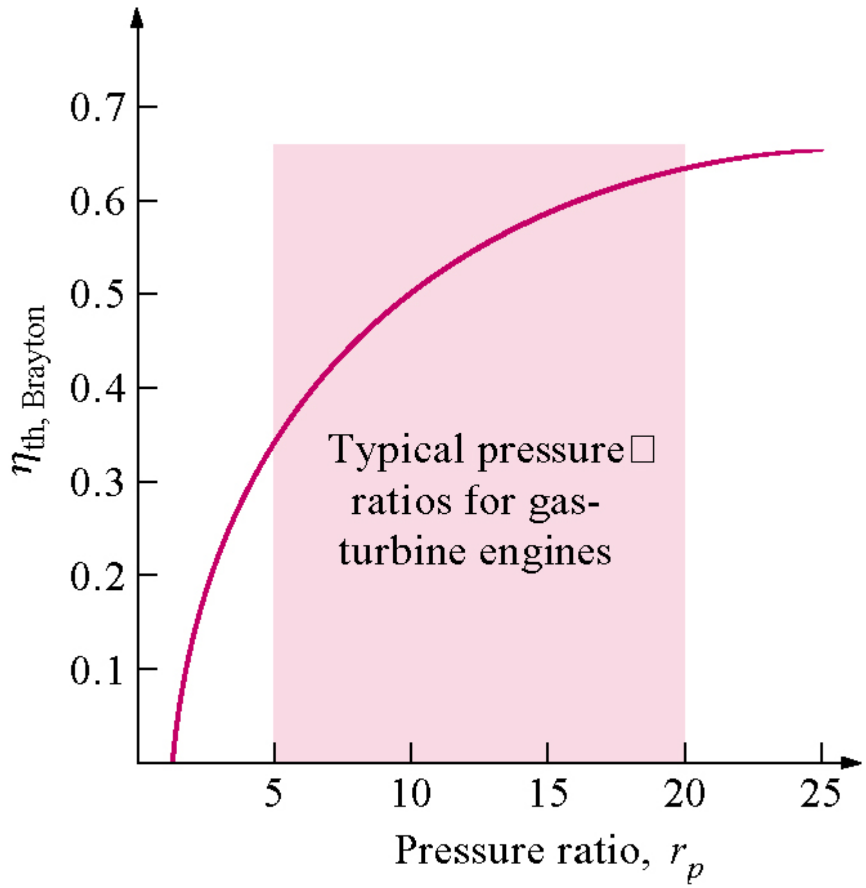
$$\frac{T_1}{T_2} = \frac{1}{r_p^{(k-1)/k}}$$

$$\eta_{th,\ Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

where the pressure ratio is $r_p = P_2/P_1$

Brayton cycle

$$\eta_{th,\ Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$



Example

The ideal air-standard Brayton cycle operates with air entering the compressor at 95 kPa, 22^{o}C. The pressure ratio r_p is 6:1 and the air leaves the heat addition process at 1100 K. Determine the compressor work and the turbine work per unit mass flow, the cycle efficiency, the back work ratio, and compare the compressor exit temperature to the turbine exit temperature. Assume constant properties.

For steady-flow and neglect changes in kinetic and potential energies to process 1-2 for the compressor. Note that the compressor is isentropic.

$$\dot{E}_{in} = \dot{E}_{out}$$

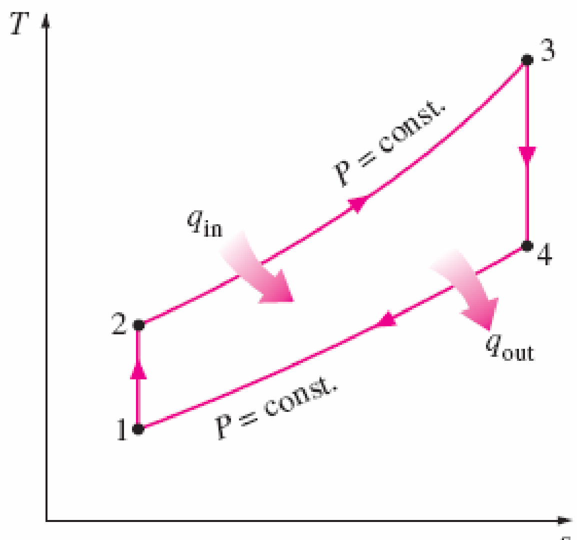
$$\dot{m}_1 h_1 + \dot{W}_{comp} = \dot{m}_2 h_2$$

The conservation of mass gives

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

For constant specific heats, the compressor work per unit mass flow is



(a) T-s diagram

$$\dot{W}_{comp} = \dot{m}(h_2 - h_1)$$

$$\dot{W}_{comp} = \dot{m}C_p(T_2 - T_1)$$

$$w_{comp} = \frac{\dot{W}_{comp}}{\dot{m}} = C_p(T_2 - T_1)$$

Since the compressor is isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = r_p^{(k-1)/k}$$

$$T_2 = T_1 r_p^{(k-1)/k}$$

$$= (22 + 273)K(6)^{(1.4-1)/1.4}$$

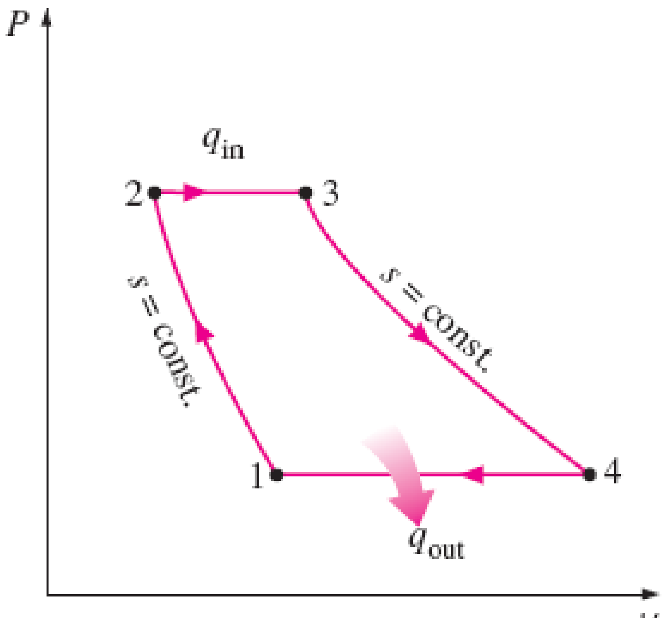
$$= 492.5\,K$$

$$w_{comp} = C_p(T_2 - T_1)$$

$$= 1.005\frac{kJ}{kg \cdot K}(492.5 - 295)K$$

$$= 198.15\frac{kJ}{kg}$$

Process 3-4; constant specific heats



(b) P-v diagram

$$\dot{W}_{turb} = \dot{m}(h_3 - h_4)$$

$$\dot{W}_{turb} = \dot{m}C_p(T_3 - T_4)$$

$$w_{turb} = \frac{\dot{W}_{turb}}{\dot{m}} = C_p(T_3 - T_4)$$

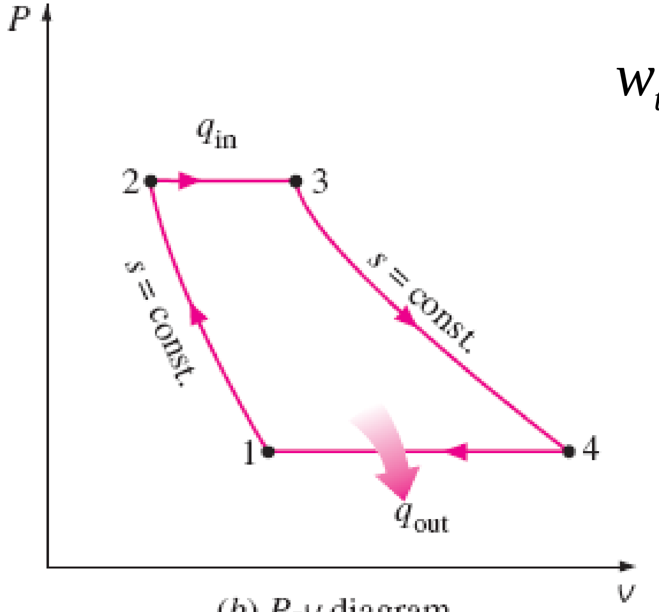
Since process 3-4 is isentropic

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k}$$

Since $P_3 = P_2$ and $P_4 = P_1$,

$$\frac{T_4}{T_3} = \left(\frac{1}{r_p}\right)^{(k-1)/k}$$

$$T_4 = T_3\left(\frac{1}{r_p}\right)^{(k-1)/k} = 1100K\left(\frac{1}{6}\right)^{(1.4-1)/1.4} = 659.1K$$



(b) P-v diagram

$$w_{turb} = C_p(T_3 - T_4) = 1.005\frac{kJ}{kg \cdot K}(1100 - 659.1)K$$

$$= 442.5\frac{kJ}{kg}$$

Process 2-3

$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

$$\dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

$$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2$$

$$= C_p(T_3 - T_2) = 1.005\frac{kJ}{kg \cdot K}(1100 - 492.5)K = 609.6\frac{kJ}{kg}$$

The net work done by the cycle is

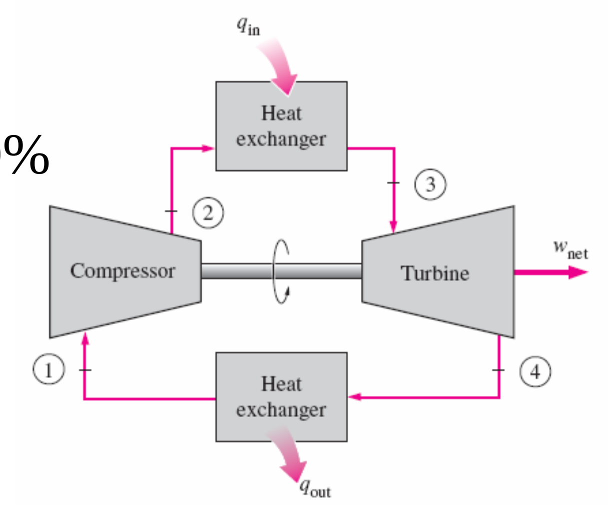
$$w_{net} = w_{turb} - w_{comp}$$

$$= (442.5 - 198.15)\frac{kJ}{kg}$$

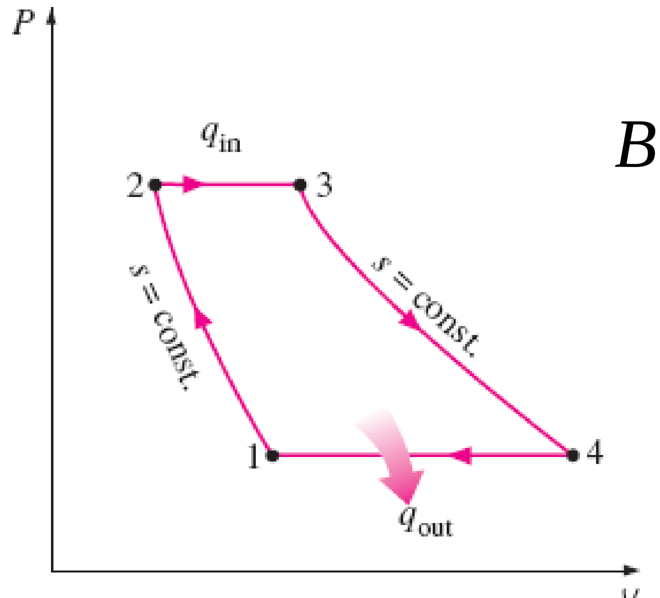
$$= 244.3\frac{kJ}{kg}$$

The cycle efficiency becomes

$$\eta_{th,\,Brayton} = \frac{w_{net}}{q_{in}} = \frac{244.3\frac{kJ}{kg}}{609.6\frac{kJ}{kg}} = 0.40 \quad or \quad 40\%$$



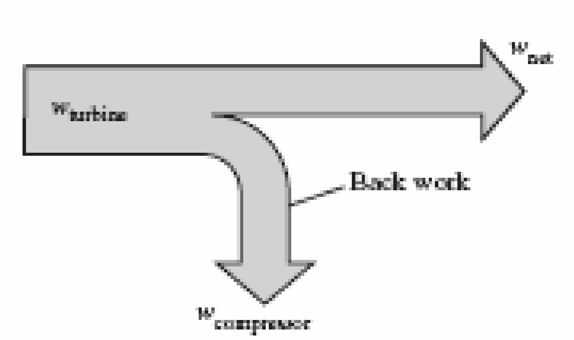
The back work ratio is defined as



(b) P-v diagram

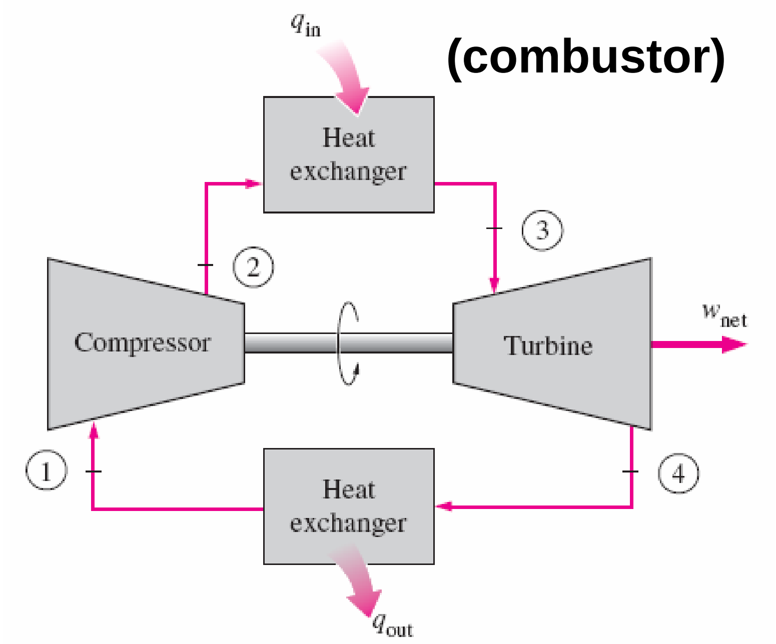
$$BWR = \frac{w_{in}}{w_{out}} = \frac{w_{comp}}{w_{turb}}$$

$$= \frac{198.15\frac{kJ}{kg}}{442.5\frac{kJ}{kg}} = 0.448$$



Note that T_4 = 659.1 K > T_2 = 492.5 K, or the turbine outlet temperature is greater than the compressor exit temperature.

What happens to η_{th}, w_{in}/w_{out}, and w_{net} as the pressure ratio r_p is increased?



$$\eta_{th,\,Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

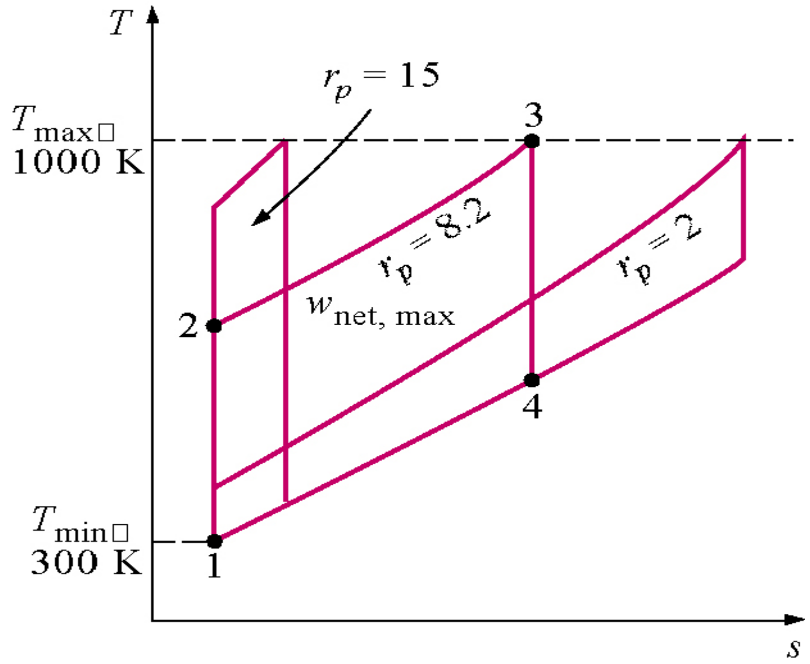
Efficiency of Brayton cycle depends on

- Specific heat ratio, k
- Pressure ratio, P_2/P_1 or r_p

Limitation in an actual gas turbine cycle

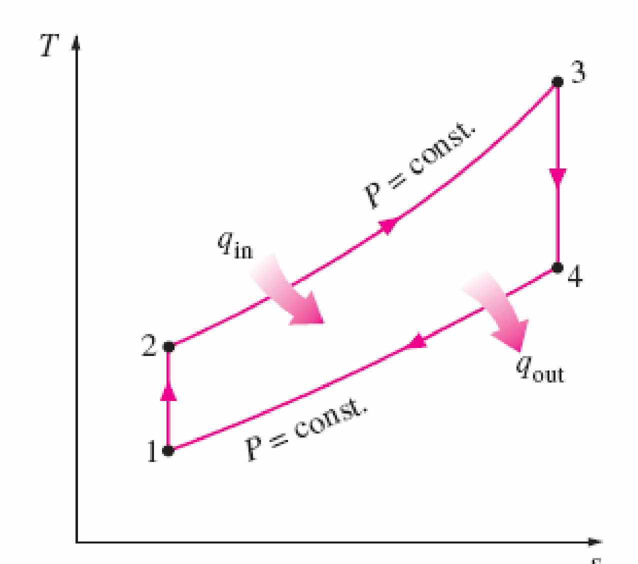
- Max. temp. is at the outlet of combustor (T_3)
  - Limitation temp. in materials

Same T_{max} and plot w_{net} in various r_p



What happens to η_{th}, w_{in} /w_{out}, and w_{net} as the pressure ratio r_p is increased?

The effect of the pressure ratio on the net work done.



(a) T-s diagram

$$w_{net} = w_{turb} - w_{comp}$$

$$= C_p(T_3 - T_4) - C_p(T_2 - T_1)$$

$$= C_p T_3 (1 - T_4 / T_3) - C_p T_1 (T_2 / T_1 - 1)$$

$$= C_p T_3 (1 - \frac{1}{r_p^{(k-1)/k}}) - C_p T_1 (r_p^{(k-1)/k} - 1)$$

Net work is **zero** when

$$r_p = 1 \quad and \quad r_p = \left(\frac{T_3}{T_1} \right)^{k/(k-1)}$$

$$w_{net} = C_p T_3 (1 - \frac{1}{r_p^{(k-1)/k}}) - C_p T_1 (r_p^{(k-1)/k} - 1)$$

- **Find Max. compression ratio**

For fixed T_3 and T_1, the pressure ratio that makes the work a maximum is obtained from:

$$\frac{dw_{net}}{dr_p} = 0$$

Let $X = r_p^{(k-1)/k}$

$$w_{net} = C_p T_3 (1 - \frac{1}{X}) - C_p T_1 (X - 1)$$

$$\frac{dw_{net}}{dX} = C_p T_3 [0 - (-1) X^{-2}] - C_p T_1 [1 - 0] = 0$$

Solving for *X*

$$X^2 = \frac{T_3}{T_1} = \left(r_p\right)^{2(k-1)/k}$$

Then, the r_p that makes the work a maximum for the constant property case and fixed T_3 and T_1 is

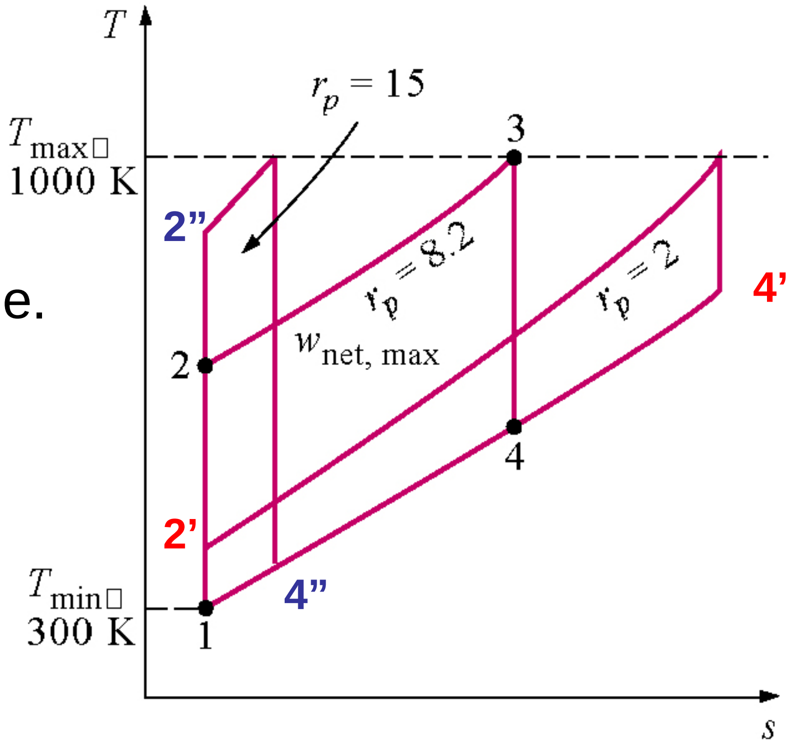
$$r_{p,\,\text{max work}} = \left(\frac{T_3}{T_1}\right)^{k/[2(k-1)]}$$

For the ideal Brayton cycle

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

And show that the following results are true.

- When $r_p = r_{p,\,\text{max work}}$, $T_4 = T_2$
- When $r_p < r_{p,\,\text{max work}}$, $T_4 > T_2$
- When $r_p > r_{p,\,\text{max work}}$, $T_4 < T_2$



Example: ideal gas turbine

A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet. Utilizing the air-standard assumptions, determine (*a*) the gas temperature at the exits of the compressor and the turbine, (*b*) the back work ratio, and (*c*) the thermal efficiency.

Air standard assumptions

- The cycles does not involve any friction. Therefore, the working fluid does not experience any pressure drop as it flow in pipe or devices such as heat exchanger.

- All expansion and compression processes take place in a quasi-equilibrium manner.

- The pipe connecting the various components of a system are well insulated, and heat transfer through them is negligible.

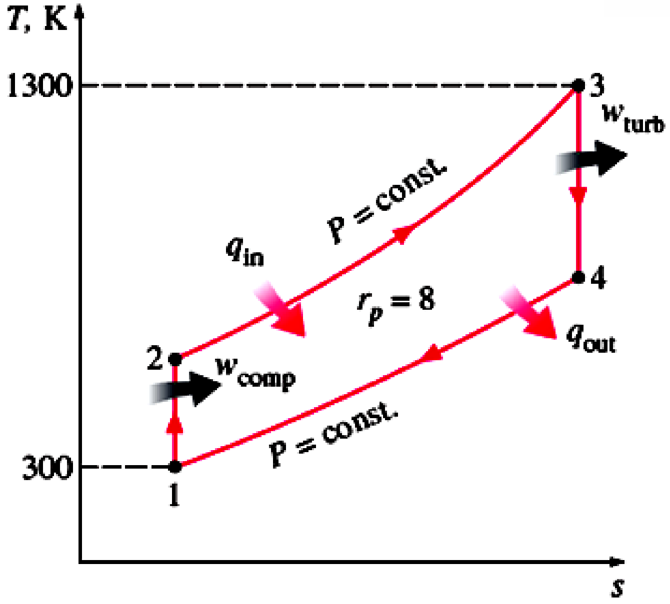
Information

Ideal Brayton cycle

Pressure ratio, $r_p = 8$

Temperature at the inlet compressor = 300 K

Temperature at the inlet turbine = 1300 K



Process 1-2 (isentropic compression of an ideal gas):

From Table A-17

$$T_1 = 300\text{ K} \rightarrow h_1 = 300.19\text{ kJ/kg}$$

$$P_{r1} = 1.386$$

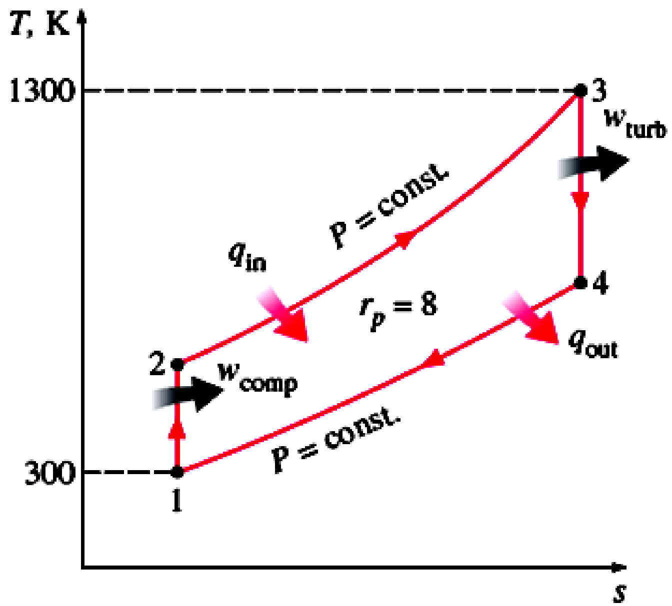
$$P_{r2} = \frac{P_2}{P_1} P_{r1} = (8)(1.386) = 11.09$$

From table and $s_1 = s_2$

$$T_2 = 540\text{ K} \quad \text{(at compressor exit)}$$

$$h_2 = 544.35\text{ kJ/kg}$$

Process 3-4 (isentropic expansion of an ideal gas):



From Table A-17

$$T_3 = 1300 \text{ K} \rightarrow h_3 = 1395.97 \text{ kJ/kg}$$

$$P_{r3} = 330.9$$

$$P_{r4} = \frac{P_4}{P_3} P_{r3} = \left(\frac{1}{8}\right)(330.9) = 41.36$$

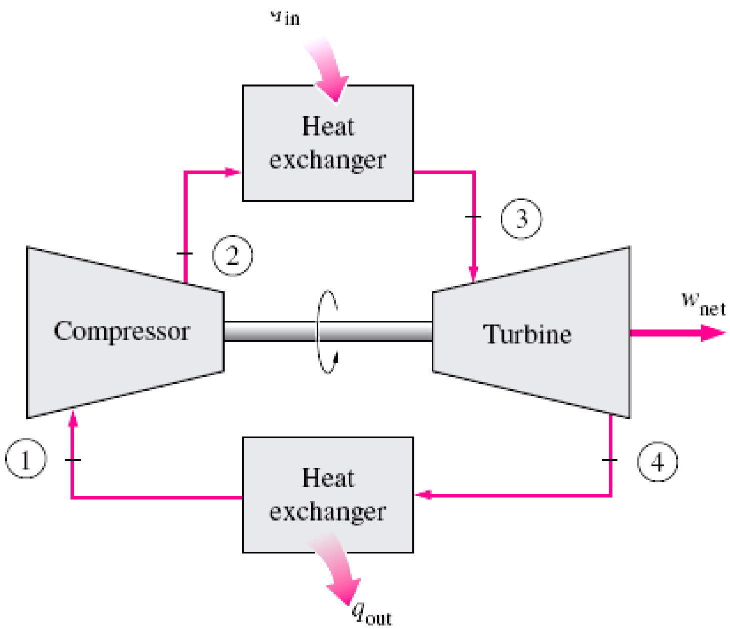
From table and $s_3 = s_4$

$$\rightarrow T_4 = 770 \text{ K} \qquad \text{(at turbine exit)}$$

$$h_4 = 789.37 \text{ kJ/kg}$$

(b) Back work ratio

$$r_{bw} = \frac{w_{comp,in}}{w_{turb,out}}$$



$$w_{\text{comp,in}} = h_2 - h_1 = 544.35 - 300.19 = 244.16 \text{ kJ/kg}$$

$$w_{\text{turb,out}} = h_3 - h_4 = 1395.97 - 789.37 = 606.60 \text{ kJ/kg}$$

$$r_{\text{bw}} = \frac{w_{\text{comp,in}}}{w_{\text{turb,out}}} = \frac{244.16 \text{ kJ/kg}}{606.60 \text{ kJ/kg}} = \mathbf{0.403}$$

(c) Thermal efficiency

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{turb,out} - w_{comp,in}}{h_3 - h_2} = \frac{362.4 \text{ kJ/kg}}{851.62 \text{ kJ/kg}} = 0.426$$

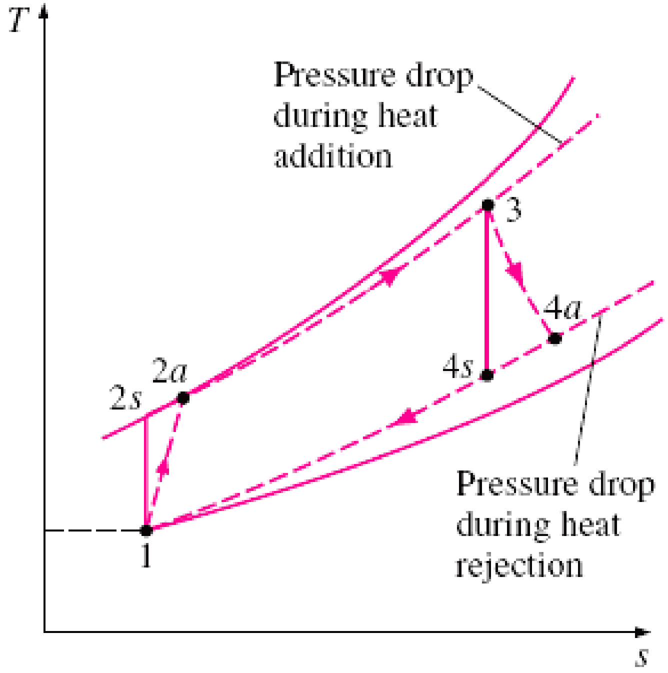
$$\eta_{\text{th,Brayton}} = 1 - \frac{1}{r_p^{(k-1)/k}} = 1 - \frac{1}{8^{(1.4-1)/1.4}} = 0.448$$

which is sufficiently close to the value obtained by accounting for the variation of specific heat with temperature

Deviation of actual gas turbine cycles from idealized ones

- ❑ Pressure drop during the heat-addition and heat rejection

- ❑ Due to irreversiblility, actual work input to the compressor is more and the actual work output from the turbine is less



Pressure & Entropy generation in actual processes

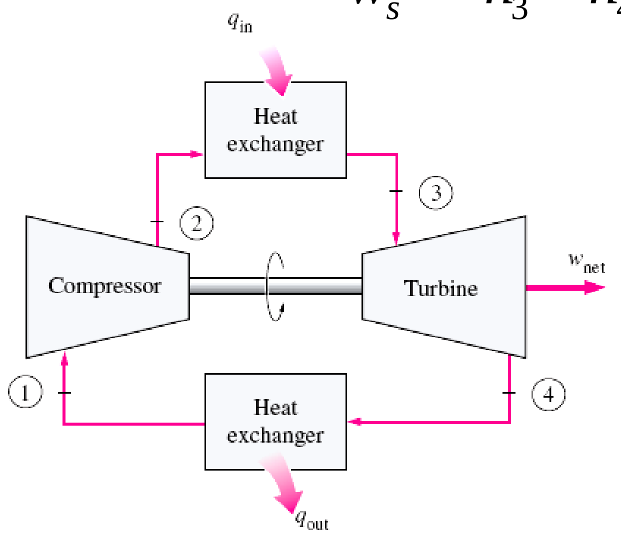
- ❑ **Isentropic efficiency**

Compressor

$$\eta_c = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

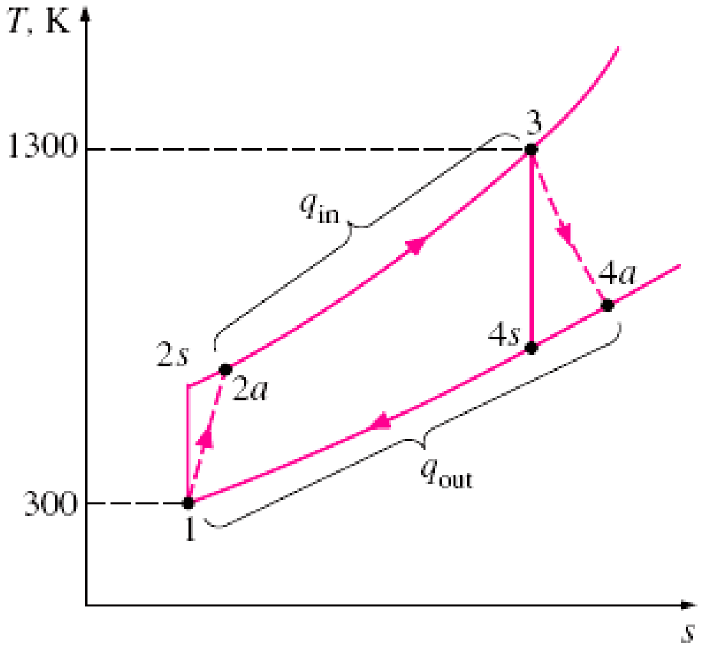
Turbine

$$\eta_T = \frac{w_a}{w_s} \cong \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$



Example: actual gas turbine

Assuming a compressor efficiency of 80 percent and a turbine efficiency of 85 percent, determine (a) the back work ratio, (b) the thermal efficiency, and (c) the turbine exit temperature of the gas turbine cycle discussed in the previous example (ideal gas turbine)



From previous example (ideal gas turbine)

$$w_{s,comp} = 244.16 \text{ kJ/kg}$$

$$w_{s,turb} = 606.6 \text{ kJ/kg}$$

$$r_{s,bw} = 0.403$$

$$\eta_{th} = 0.426$$

For actual gas turbine

Compressor

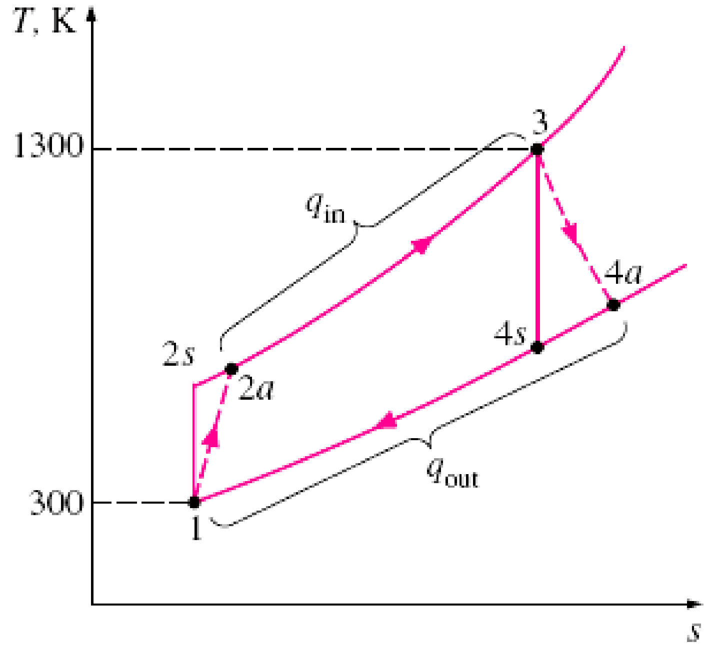
$$w_{comp,in} = \frac{w_{s,comp}}{\eta_c} = \frac{244.16 \text{ kJ/kg}}{0.8} = 305.2 \text{ kJ/kg}$$

turbine

$$w_{turb,out} = \eta_T w_{s,turb} = (0.85)(606.6 \text{ kJ/kg}) = 515.61 \text{ kJ/kg}$$

Back work ratio

$$r_{bw} = \frac{w_{comp,in}}{w_{turb,out}} = \frac{305.2}{515.61} = 0.592$$



back work ratio in actual case is larger than that in ideal case, where $r_{s,bw}$ = 0.403

(b) Thermal efficiency

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{turb,out} - w_{comp,in}}{h_3 - h_{2a}} = \frac{515.61 - 605.39}{1395.97 - 605.36} = 0.266$$

$$w_{comp,in} = h_{2a} - h_1 \quad \rightarrow h_{2a} = h_1 + w_{comp,in}$$

$$= 300.19 + 305.2 = 605.36\ kJ/kg$$

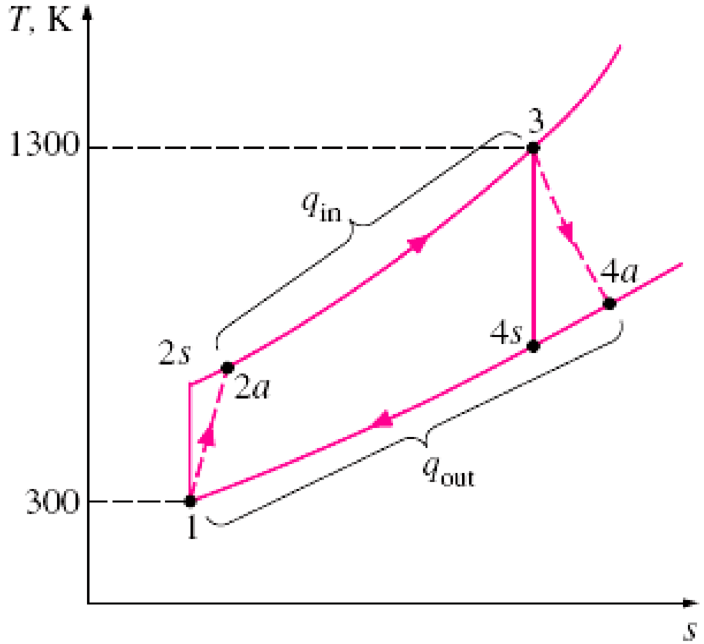
Thermal efficiency in actual case is lower than that in ideal case, where η_{th} = 0.426

(c) Air temperature at the turbine exit

Apply energy balance on turbine

$$w_{turb,out} = h_3 - h_{4a} \rightarrow h_{4a} = h_3 - w_{turb,out}$$

$$h_{4a} = 1395.97 - 515.61 = 880.36\ kJ/kg$$

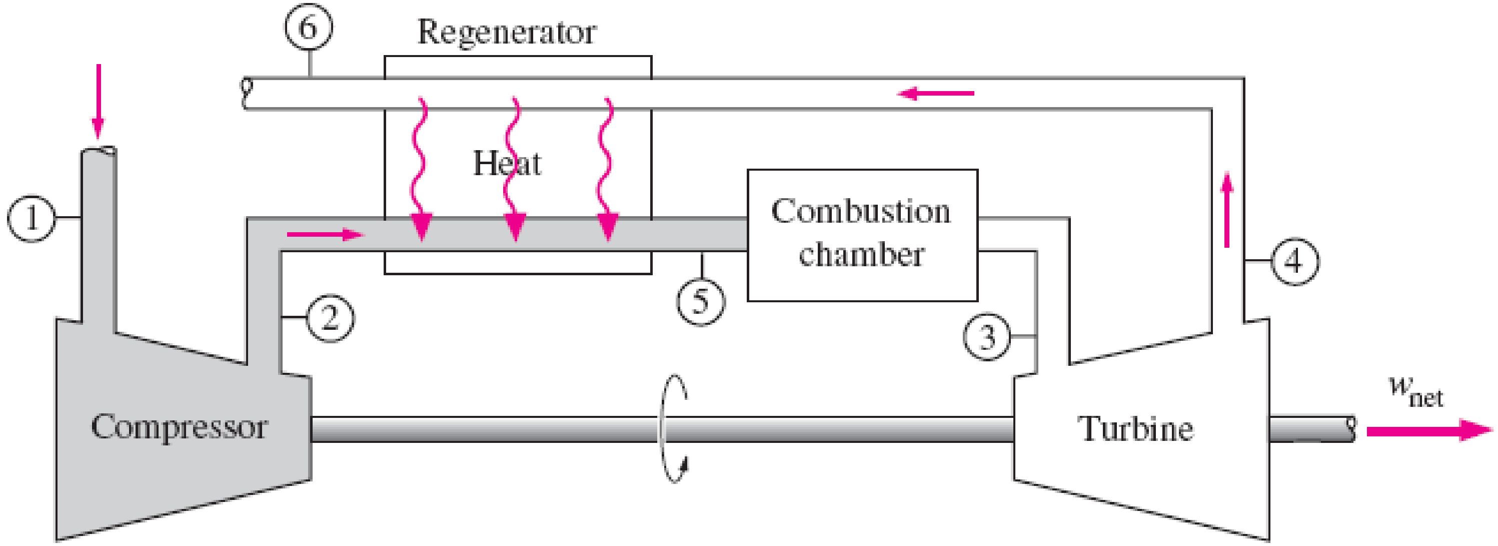


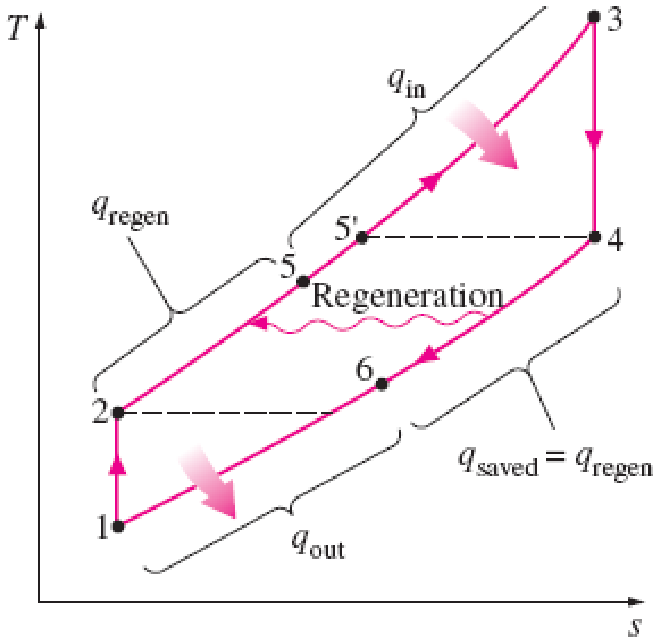
Find T_{4a} from Table

T_{4a} = 853 K

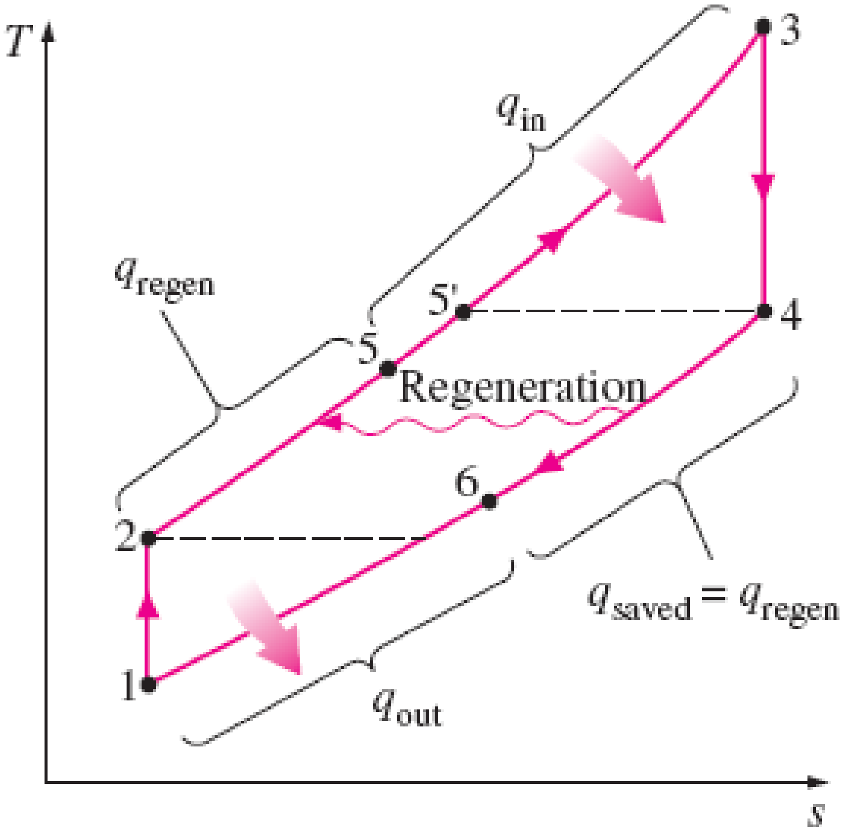
The temperature at turbine exit is considerably higher than that at the compressor exit (T2a = 598 K), which suggests the use of regeneration to reduce fuel cost

The Brayton cycle with regeneration





- Temperature of the exhaust gas from turbine is higher than the temperature of the air leaving the compressor.
- Use regenerator or recuperator as an heat exchanger



- Actual heat transfer from exhaust gas to the air

$$q_{regen,act} = h_5 - h_2$$

- Max. heat transfer from exhaust gas to the air

$$q_{regen,\max} = h_{5'} - h_2 = h_4 - h_2$$

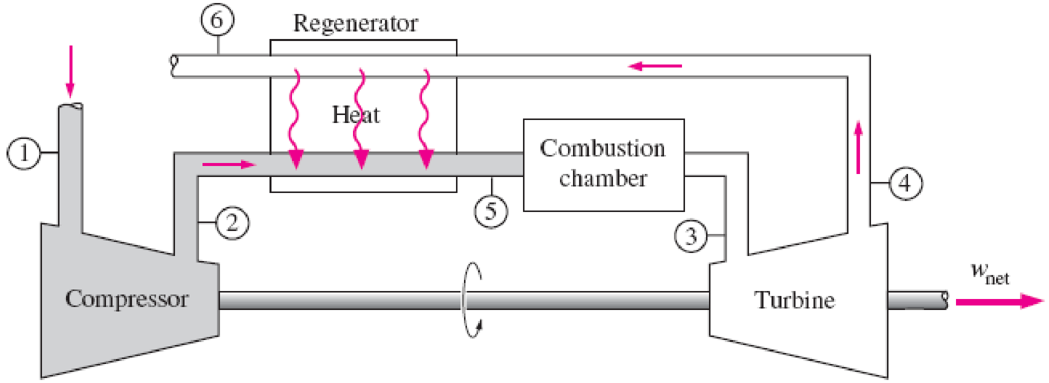
T_4 is temp from exhaust turbine gas

- Extent to which a regenerator approaches an ideal regenerator is called **the effectiveness ε,**

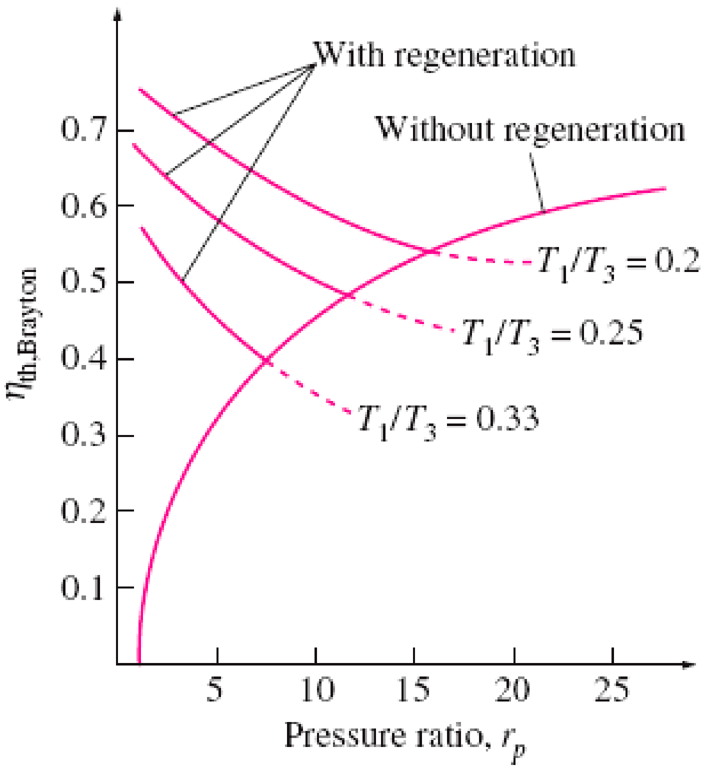
$$\varepsilon = \frac{q_{regen,act}}{q_{regen,\max}} = \frac{h_5 - h_2}{h_4 - h_2}$$

• For the cold-air-standard assumptions are utilized, it reduces to

$$\varepsilon \approx \frac{T_5 - T_2}{T_4 - T_2}$$



• Higher effectiveness requires the use of larger regenerator



• Due to higher price of components and larger pressure drop, the use of larger regenerator with very high effectiveness does not justify saving money

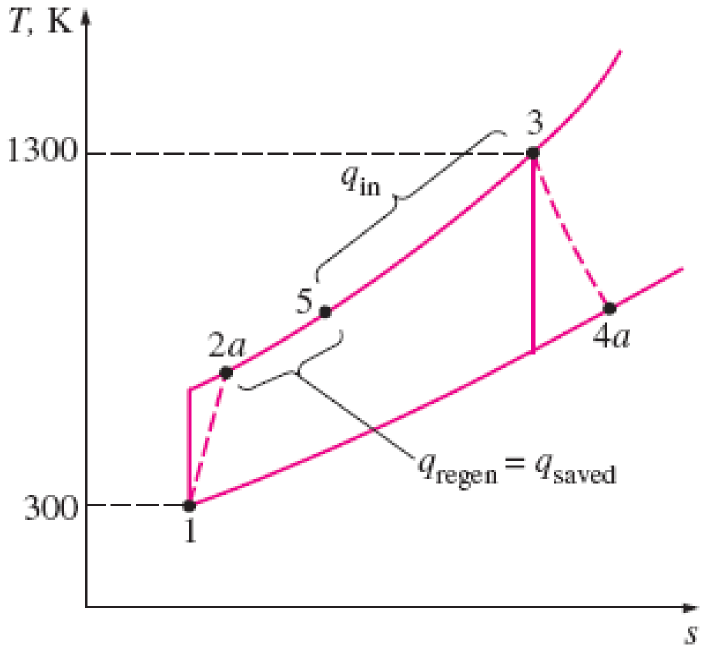
• Thermal efficiency of an ideal Brayton cycle with regenerator

$$\eta_{th,regen} = 1 - \left(\frac{T_1}{T_3}\right)\left(r_p\right)^{\frac{k-1}{k}}$$

• T_3 is T_{max}, and T_1 is T_{min}

Example

Use the information from previous example (actual gas turbine), and determine the thermal efficiency if a regenerator having an effectiveness of 80 percent is used.



The effectiveness

$$\varepsilon = \frac{q_{regen,act}}{q_{regen,\max}} = \frac{h_5 - h_{2a}}{h_{4a} - h_{2a}}$$

$$0.8 = \frac{h_5 - 605.39}{880.36 - 605.39}$$

$$\rightarrow h_5 = 825.37\ kJ/kg$$

$$q_{in} = h_3 - h_5 = 1395.97 - 825.37 = 570.6\ kJ/kg$$

Thus a saving of 220 kJ/kg from the heat input requirement, i.e., 970.61-570.6 kJ/kg

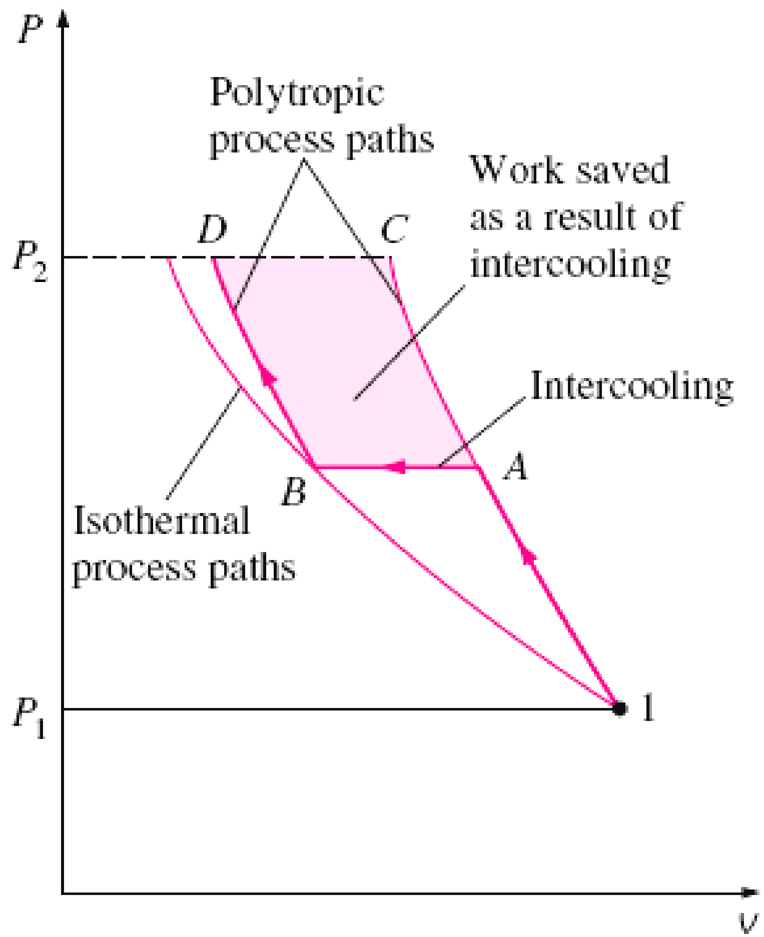
With regenerator

$$\eta_{th,regen} = \frac{w_{net}}{q_{in}} = \frac{210.41}{570.6} = 0.369$$

Without regenerator

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 0.266$$

Brayton cycle with intercooling, reheating, and regeneration



$$w_{net} = w_{turb,out} - w_{comp,in}$$

- Work required to compress a gas between two specified pressure can be decreased by carrying out the compression process in stages and cooling gas, i.e., multi-stage compression with *intercooling.*

Comparison of work inputs to a single-stage compressor (1AC) and a two-stage compressor with intercooling (1ABD)

Intercooling

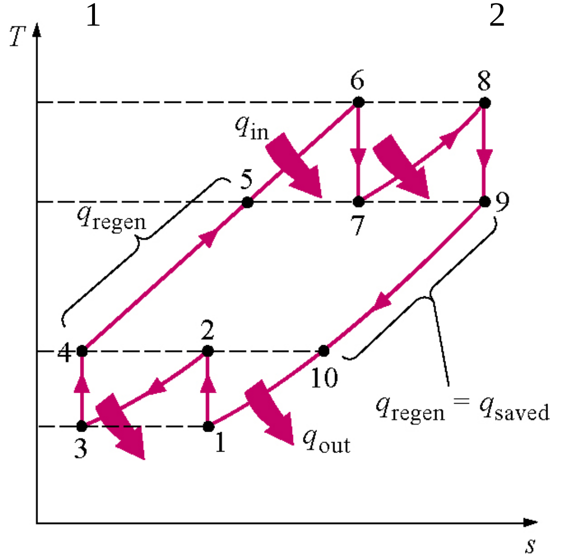
When using multistage compression, cooling the working fluid between the stages will reduce the amount of compressor work required. The compressor work is reduced because cooling the working fluid reduces the average specific volume of the fluid and thus reduces the amount of work on the fluid to achieve the given pressure rise.

$$\int v dP$$

To determine the intermediate pressure at which intercooling should take place to minimize the compressor work

$$w_{comp} = \int_1^4 v\, dP = \int_1^2 v\, dP + \int_2^3 \cancelto{0}{v\, dP} + \int_3^4 v\, dP$$

For the adiabatic, steady-flow compression process, the work input to the compressor per unit mass is



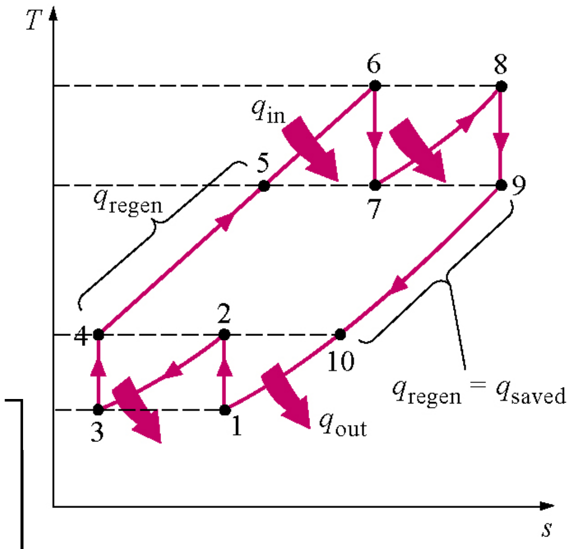
For the isentropic compression process

$$w_{comp} = \frac{k}{k\text{-}1}(P_2v_2 - P_1v_1) + \frac{k}{k\text{-}1}(P_4v_4 - P_3v_3)$$

$$= \frac{k}{k\text{-}1}R(T_2 - T_1) + \frac{kR}{k\text{-}1}(T_4 - T_3)$$

$$= \frac{k}{k\text{-}1}R\left[T_1(T_2/T_1 - 1) + T_3(T_4/T_3 - 1)\right]$$

$$= \frac{k}{k\text{-}1}R\left[T_1\left(\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right) + T_3\left(\left(\frac{P_4}{P_3}\right)^{(k-1)/k} - 1\right)\right]$$



Notice that the fraction $kR/(k\text{-}1) = C_p$.

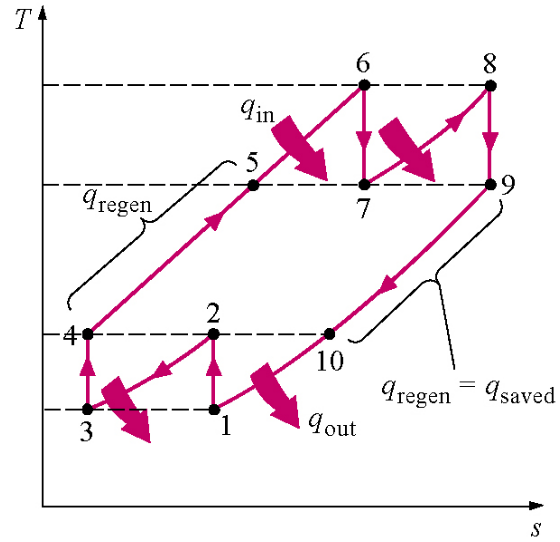
$$w_{comp} = C_p\left[T_1\left(\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right) + T_3\left(\left(\frac{P_4}{P_3}\right)^{(k-1)/k} - 1\right)\right]$$

For two-stage compression, let's assume that intercooling takes place at constant pressure and the gases can be cooled to the inlet temperature for the compressor, such that $P_3 = P_2$ and $T_3 = T_1$.

The total work supplied to the compressor becomes

$$w_{comp} = C_p T_1 \left[\left(\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right) + \left(\left(\frac{P_4}{P_2} \right)^{(k-1)/k} - 1 \right) \right]$$

$$= C_p T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} + \left(\frac{P_4}{P_2} \right)^{(k-1)/k} - 2 \right]$$

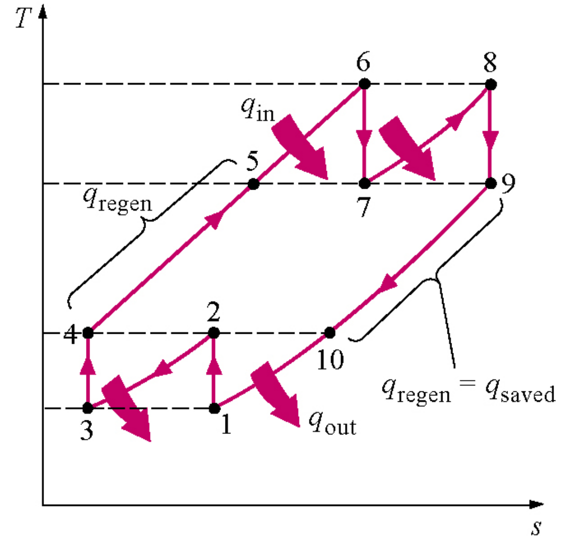


To find the unknown pressure P_2 that gives the minimum work input for fixed compressor inlet conditions T_1, P_1, and exit pressure P_4, we set

$$\frac{dw_{comp}(P_2)}{dP_2} = 0$$

This yields

$$P_2 = \sqrt{P_1 P_4}$$

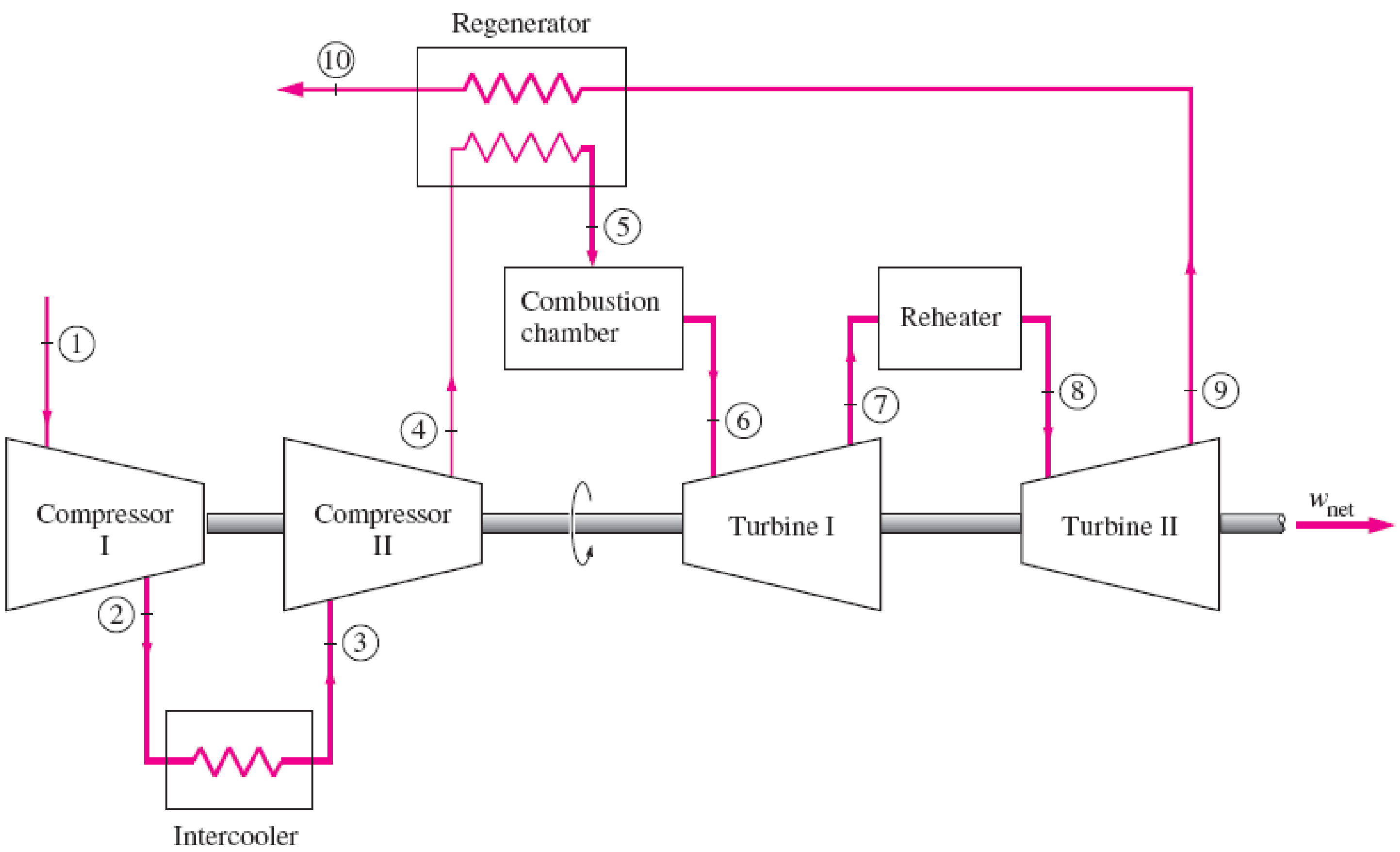


or, the pressure ratios across the two compressors are equal.

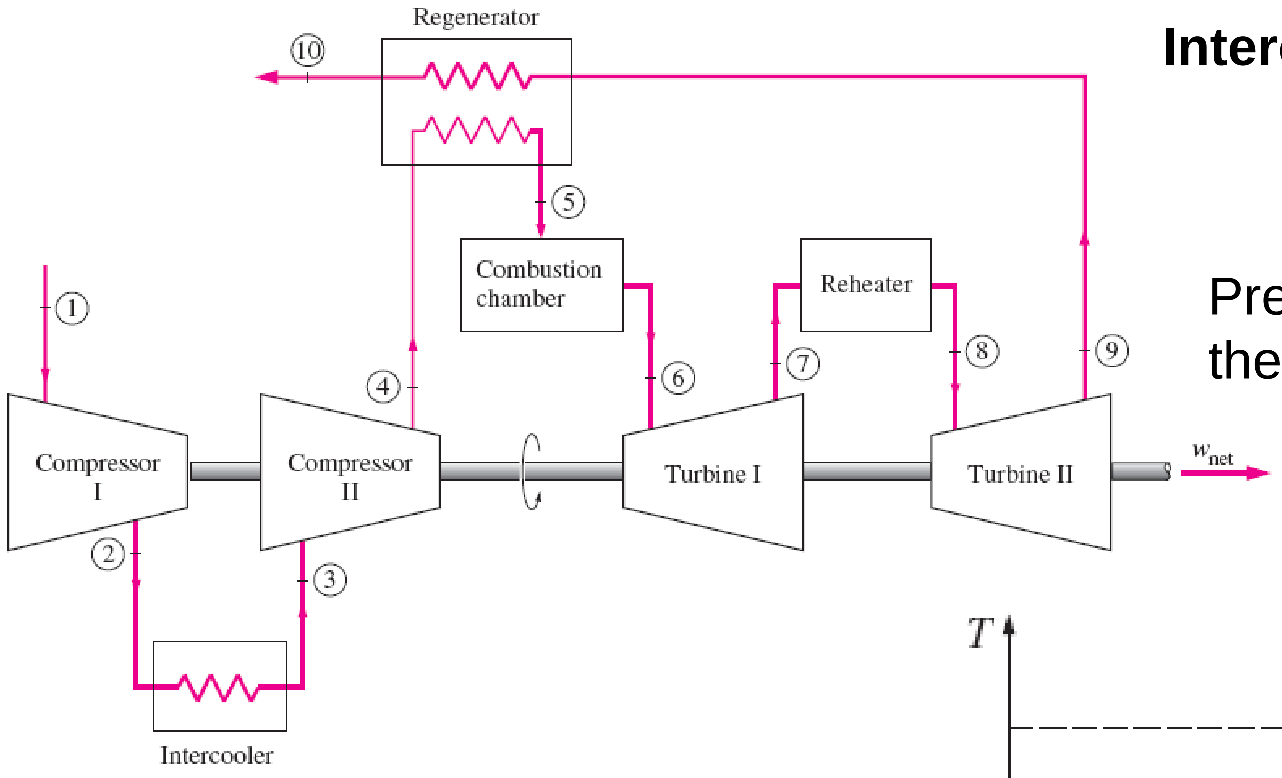
$$\frac{P_2}{P_1} = \frac{P_4}{P_2} = \frac{P_4}{P_3}$$

Intercooling is almost always used with regeneration. During intercooling the compressor final exit temperature is reduced; therefore, more heat must be supplied in the heat addition process to achieve the maximum temperature of the cycle. Regeneration can make up part of the required heat transfer.

A gas-turbine engine with two-stage compression with intercooling, two-stage expansion with reheating, and regeneration



- Work output from turbine can be increased by multi-stage expansion with reheating.



Intercooling

$$P_2 = \sqrt{P_1 P_4}$$

Pressure ratios across the two compressors

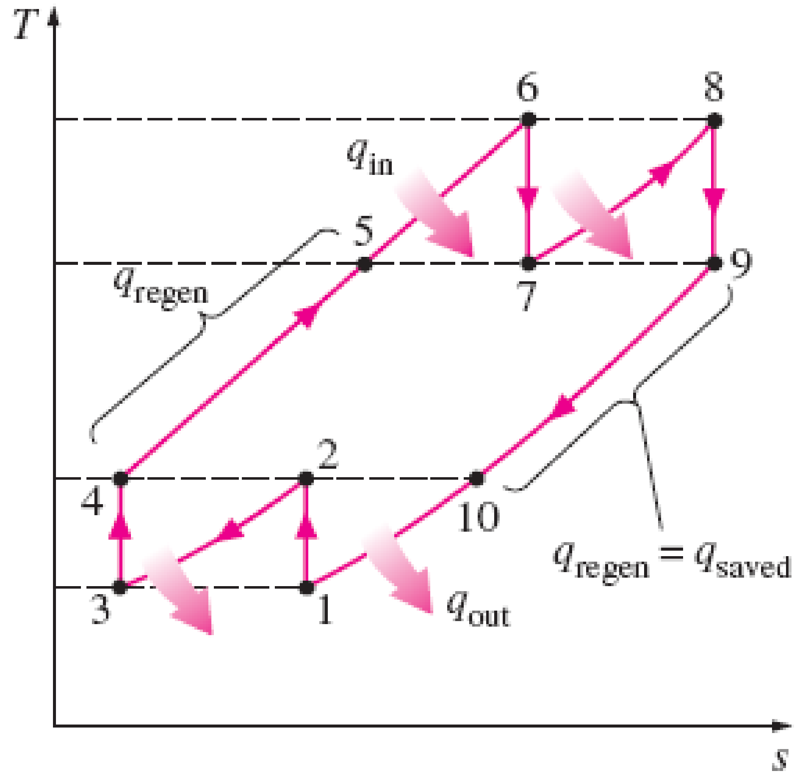
$$\frac{P_2}{P_1} = \frac{P_4}{P_2} = \frac{P_4}{P_3}$$

Reheating

$$P_7 = \sqrt{P_6 P_9}$$

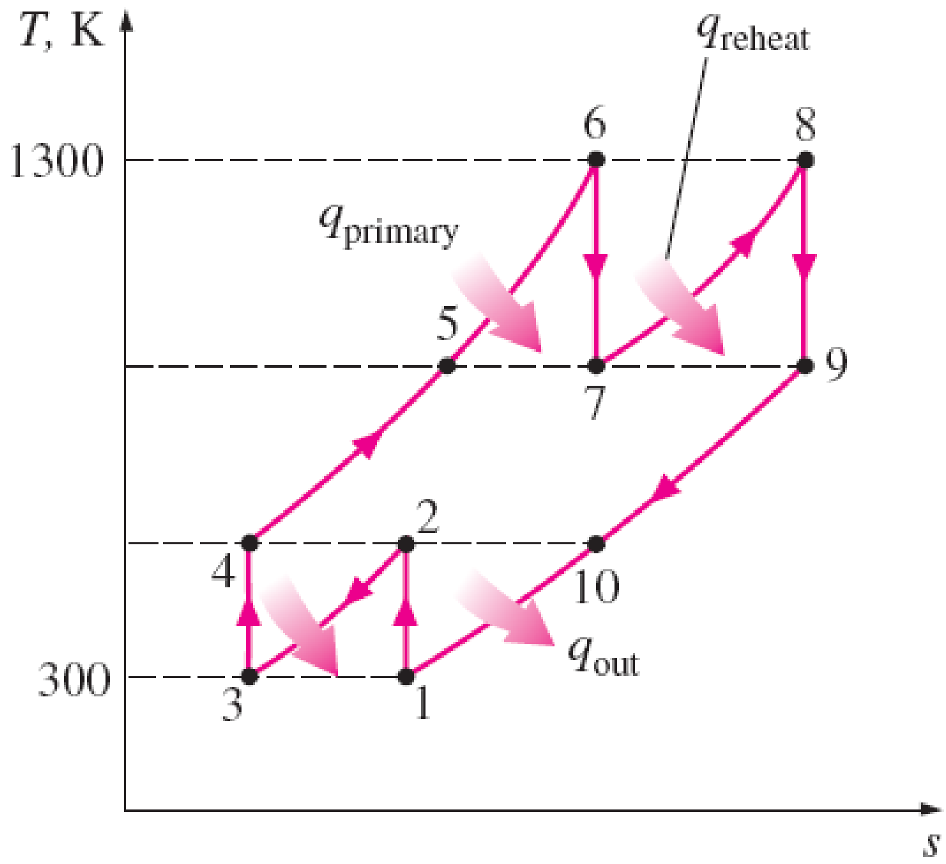
or the pressure ratios across the two turbines are equal.

$$\frac{P_6}{P_7} = \frac{P_7}{P_9} = \frac{P_8}{P_9}$$



Example

An ideal gas-turbine cycle with two stages of compression and two stages of expansion has an overall pressure ratio of 8. Air enters each stages of the compressor at 300 K and each stage of the turbine at 1300 K. Determine the back work ratio and the thermal efficiency of this gas-turbine cycle, assuming (a) no regenerators and (b) an ideal regenerator with 100 percent effectiveness.



Assumptions

- Assume steady operating conditions
- Use air-standard assumptions
- Neglect KE & PE

Information

Overall pressure ratio of 8

- each stage of compressor and each stage of turbine have same pressure ratio

$$\frac{P_2}{P_1} = \frac{P_4}{P_3} = \sqrt{8} = 2.83$$

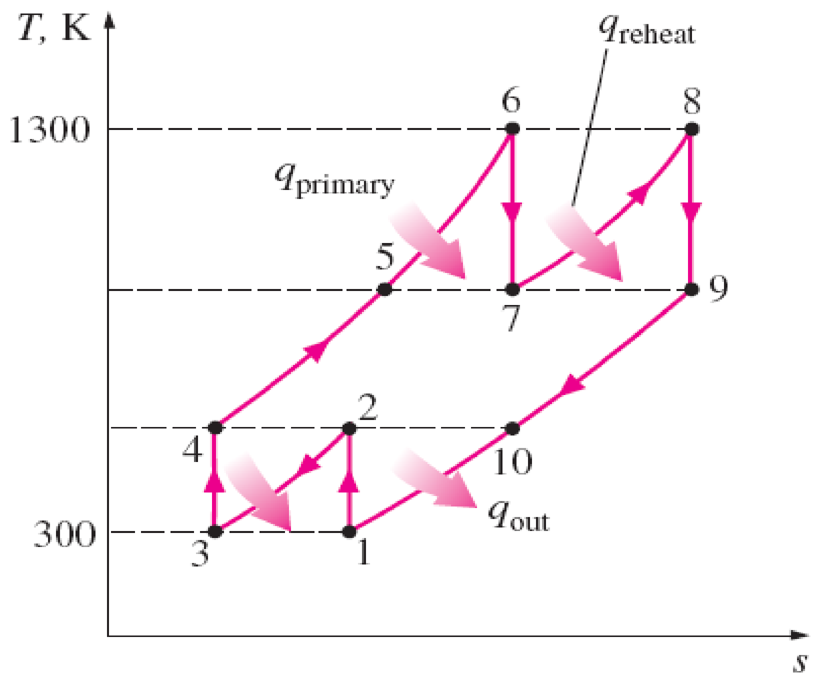
At turbine $$\frac{P_6}{P_7} = \frac{P_8}{P_9} = \sqrt{8} = 2.83$$

Air enters each stage of the compressor at the same temperature, and each stage has the same isentropic efficiency (100 percent in this case). Therefore, the temperature (and enthalpy) of the air at the exit of each compression stage will be the same. A similar argument can be given for the turbine.

At inlets: $T_1 = T_3, \quad h_1 = h_3$ and $T_6 = T_8, \quad h_6 = h_8$

At exits: $T_2 = T_4, \quad h_2 = h_4$ and $T_7 = T_9, \quad h_7 = h_9$

Under these conditions, the work input to each stage of the compressor will be same



(a) No regeneration case

Determine the back work ratio & thermal efficiency

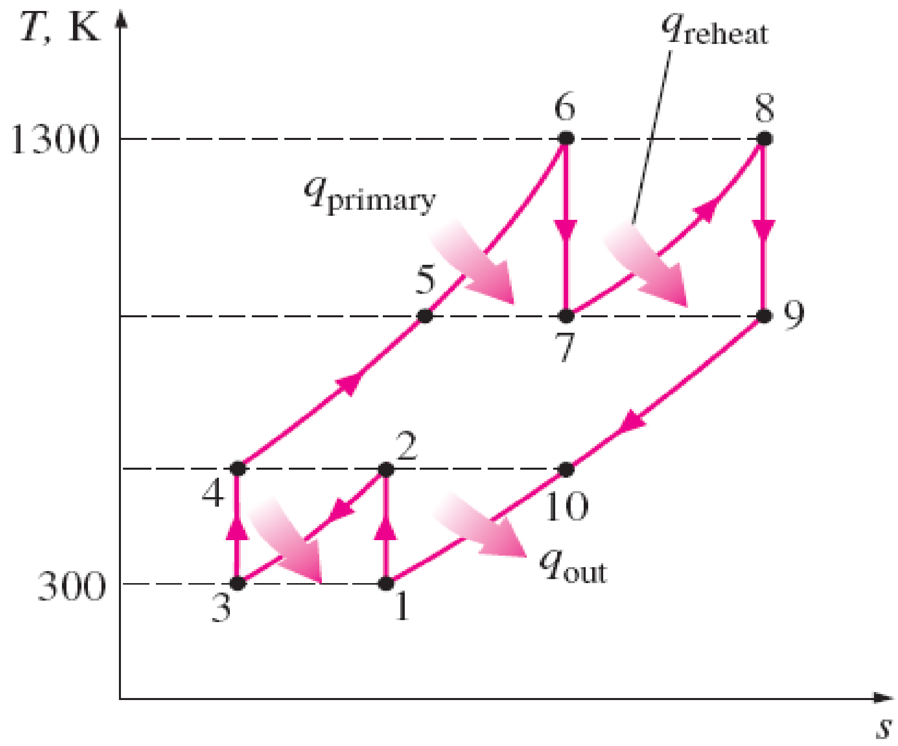
$$r_{bw} = \frac{w_{comp,in}}{w_{turb,out}} \qquad \eta_{th} = \frac{w_{net}}{q_{in}}$$

$$w_{net} = w_{turb,out} - w_{comp,in}$$

$$w_{comp,in} = 2(w_{comp,in,I}) = 2(h_2 - h_1)$$

$$w_{turb,out} = 2(w_{turb,out,I}) = 2(h_6 - h_7)$$

$$q_{in} = q_{primary} + q_{reheat} = (h_6 - h_4) + (h_8 - h_7)$$



From table A-17

$$T_1 = 300\text{ K} \rightarrow h_1 = 300.19\text{ kJ/kg}$$

$$P_{r1} = 1.386$$

$$P_{r2} = \frac{P_2}{P_1}P_{r1} = \sqrt{8}(1.386) = 3.92 \rightarrow T_2 = 403.3\text{ K}$$

$$h_2 = 404.31\text{ kJ/kg}$$

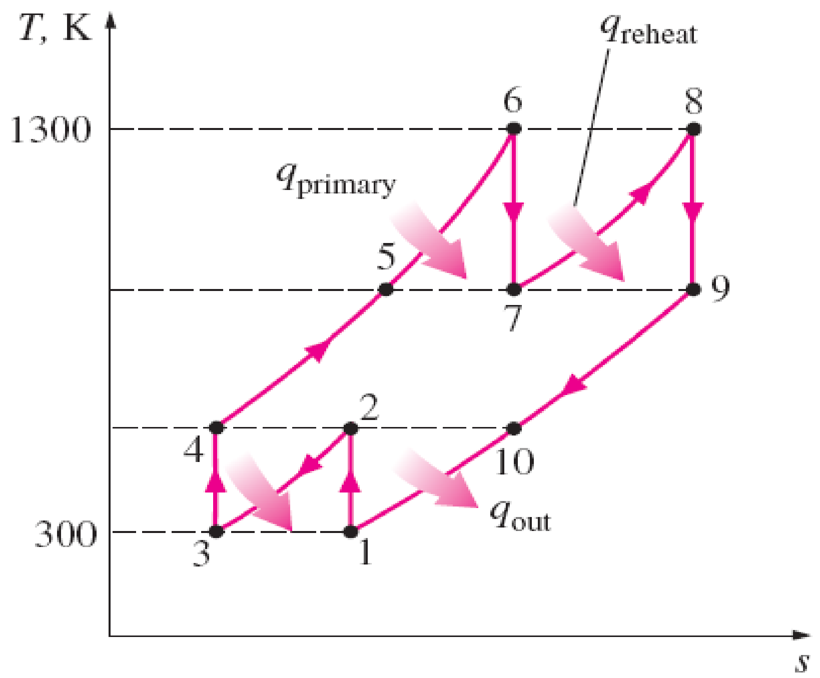
$$T_6 = 1300\text{ K} \rightarrow h_6 = 1395.97\text{ kJ/kg}$$

$$P_{r6} = 330.9$$

$$P_{r7} = \frac{P_7}{P_6}P_{r6} = \frac{1}{\sqrt{8}}(330.9)$$

$$= 117.0 \rightarrow T_7 = 1006.4\text{ K}$$

$$h_7 = 1053.33\text{ kJ/kg}$$

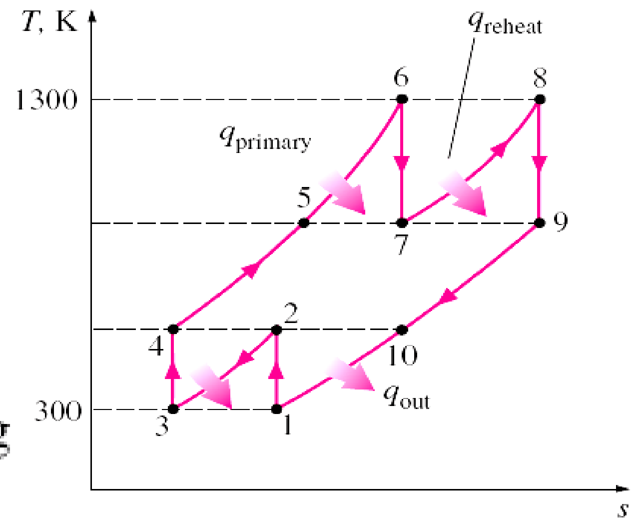


$$w_{comp,in} = 2(w_{comp,in,I}) = 2(h_2 - h_1)$$

$$= 2(404.31 - 300.19) = 208.24 \text{ kJ/kg}$$

$$w_{turb,out} = 2(w_{turb,out,I}) = 2(h_6 - h_7)$$

$$= 2(1395.97 - 1053.33) = 685.28 \text{ kJ/kg}$$



$$w_{net} = w_{turb,out} - w_{comp,in}$$

$$= 685.28 - 208.24 = 477.04 \text{ kJ/kg}$$

$$q_{in} = q_{primary} + q_{reheat} = (h_6 - h_4) + (h_8 - h_7)$$

$$= (1395.97 - 404.31) + (1395.97 - 1053.33) = 1334.30 \text{ kJ/kg}$$

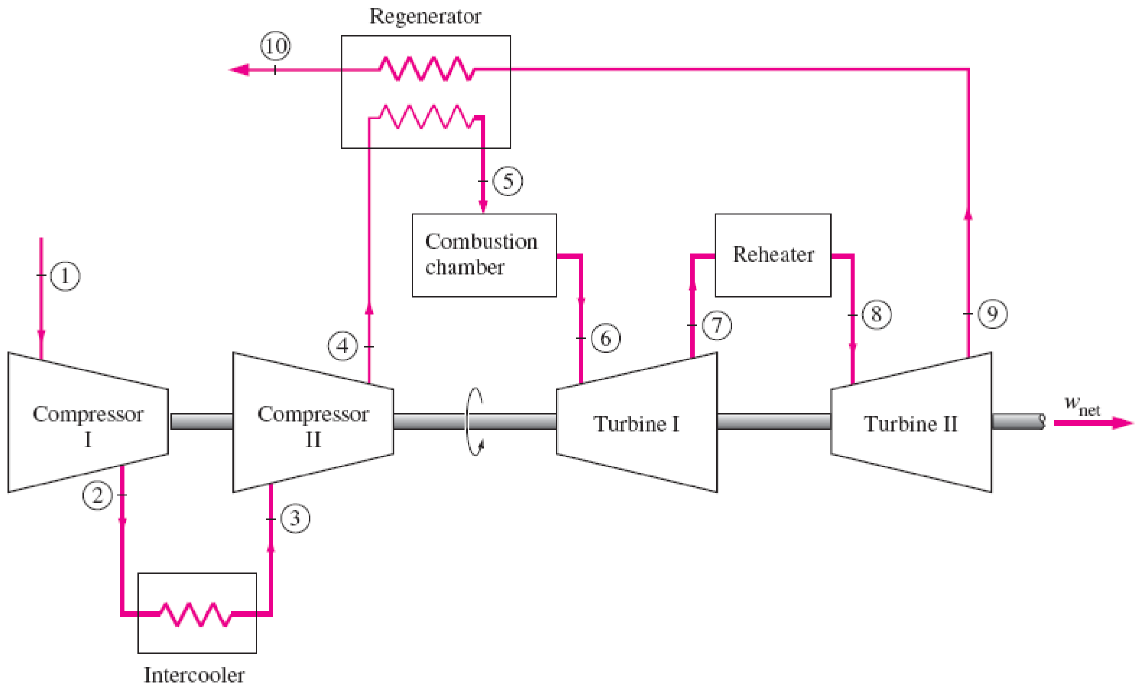
$$r_{bw} = \frac{w_{comp,in}}{w_{turb,out}} = \frac{208.24 \text{ kJ/kg}}{685.28 \text{ kJ/kg}} = \mathbf{0.304 \text{ or } 30.4\%}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{477.04 \text{ kJ/kg}}{1334.30 \text{ kJ/kg}} = \mathbf{0.358 \text{ or } 35.8\%}$$

(b) Add an ideal regenerator (no pressure drop, 100 percent effectiveness)

Compressed air is heated to the turbine exit temperature T_9, before it enters the combustion chamber.

Under the air-standard assumptions, $h_5 = h_7 = h_9$



$$q_{in} = q_{primary} + q_{reheat} = (h_6 - h_5) + (h_8 - h_7)$$

$$= (1395.97 - 1053.33) + (1395.97 - 1053.33) = 685.28 \text{ kJ/kg}$$

Two stage compression and expansion with intercooling, reheating, and regeneration

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{477.04 \text{ kJ/kg}}{685.28 \text{ kJ/kg}} = \mathbf{0.696 \text{ or } 69.6\%}$$

Two stage compression and expansion with intercooling, reheating (without regeneration)

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{477.04 \text{ kJ/kg}}{1334.30 \text{ kJ/kg}} = \mathbf{0.358 \text{ or } 35.8\%}$$