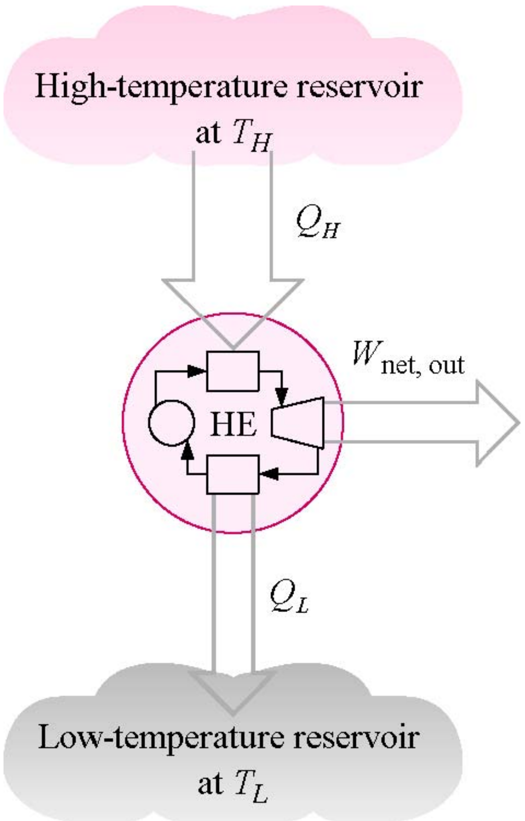
Vapor and Combined Power Cycles

Vapor power cycles

- ❑ A gas power cycle considers air as the working fluid $\rightarrow$ single phase
- ❑ A vapor power cycle considers steam as the working fluid $\rightarrow$ might be two phases

Carnot Vapor Cycle



- ❑ The use of steam as working fluid
  - Low cost
  - Availability
  - High enthalpy of vaporization

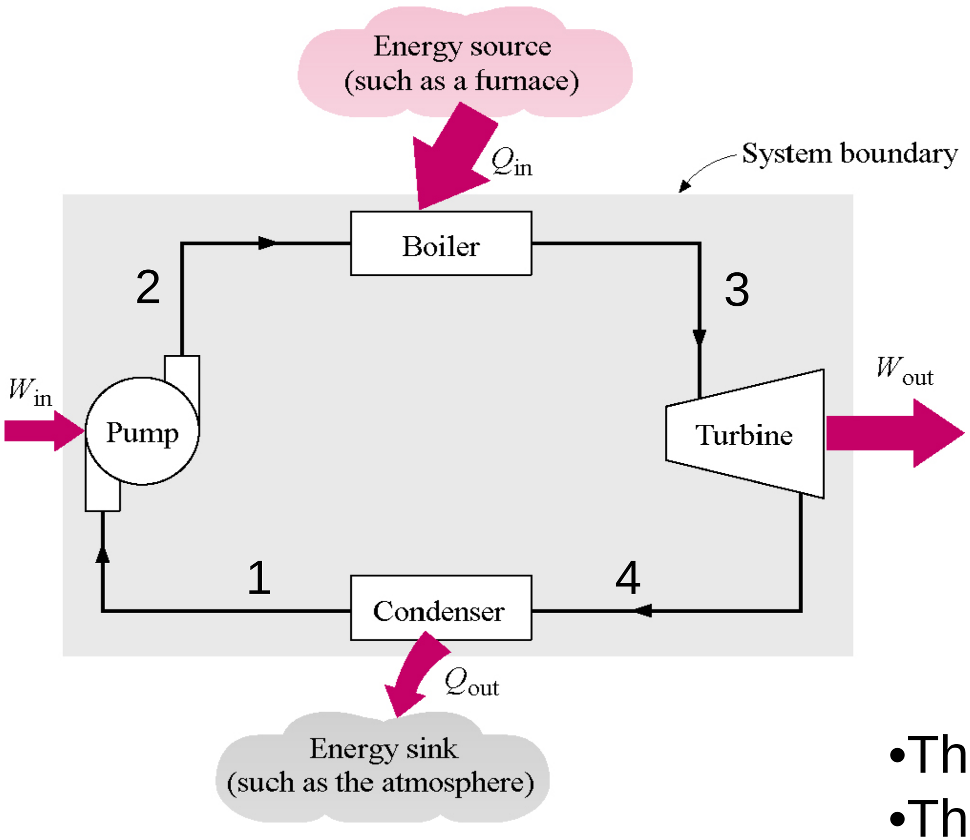
- ❑ Carnot cycle
  - Give maximum thermal efficiency
  - But not be suitable to use for analyzing the efficiency of vapor power cycle

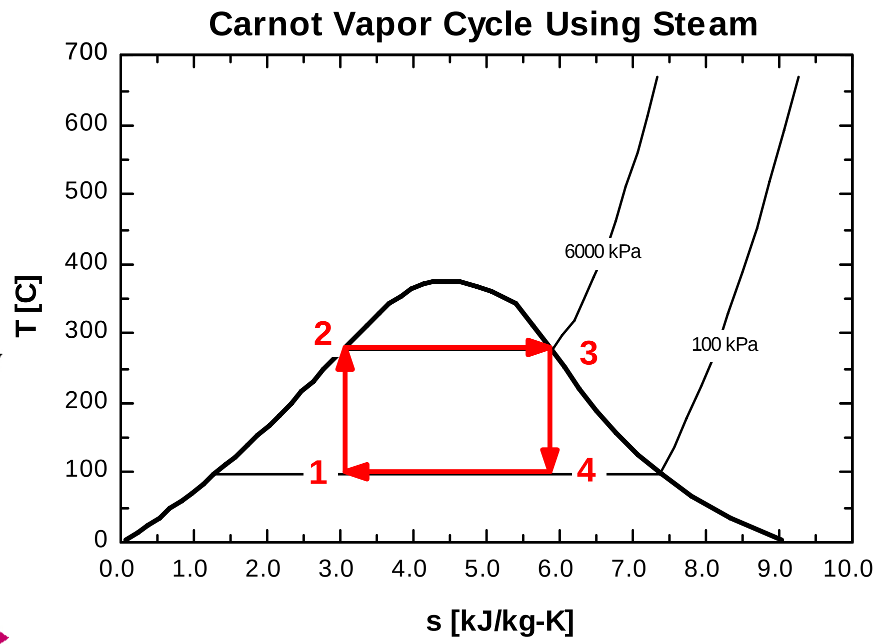
Objectives

- Analyze vapor power cycles in which the working fluid is alternately vaporized and condensed.
- Analyze power generation coupled with process heating, called cogeneration.
- Investigate methods to modify the basic Rankine vapor power cycle to increase the cycle thermal efficiency.
- Analyze the reheat and regenerative vapor power cycles.
- Analyze power cycles that consist of two separate cycles known as combined cycles and binary cycles.

Carnot vapor cycle

The working fluid, steam (water), undergoes a thermodynamic cycle from 1-2-3-4-1.





$$\eta_{th,\,Carnot} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_L}{T_H}$$

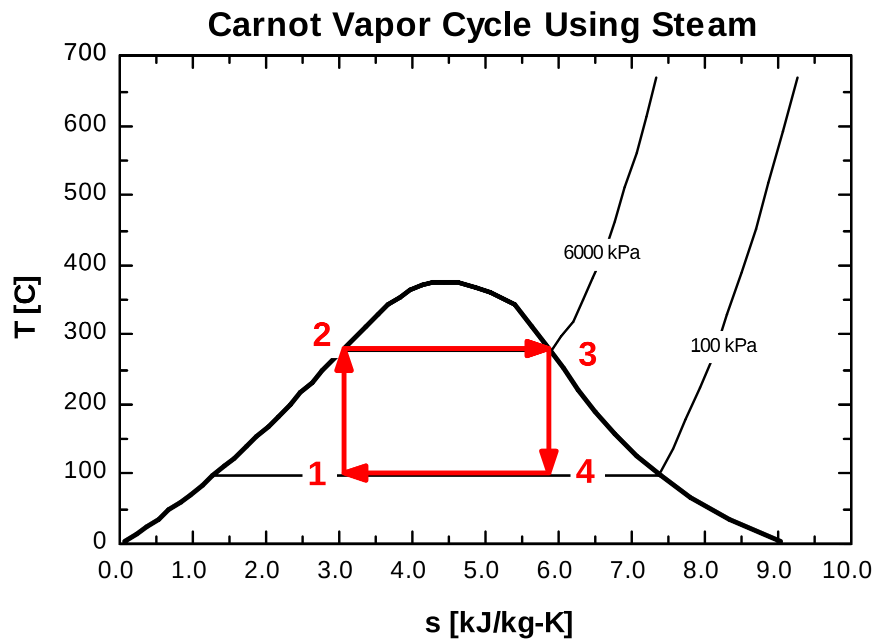
- The larger the T_H the larger the $\eta_{th,\ Carnot}$
- The smaller the T_L the larger the $\eta_{th,\ Carnot}$

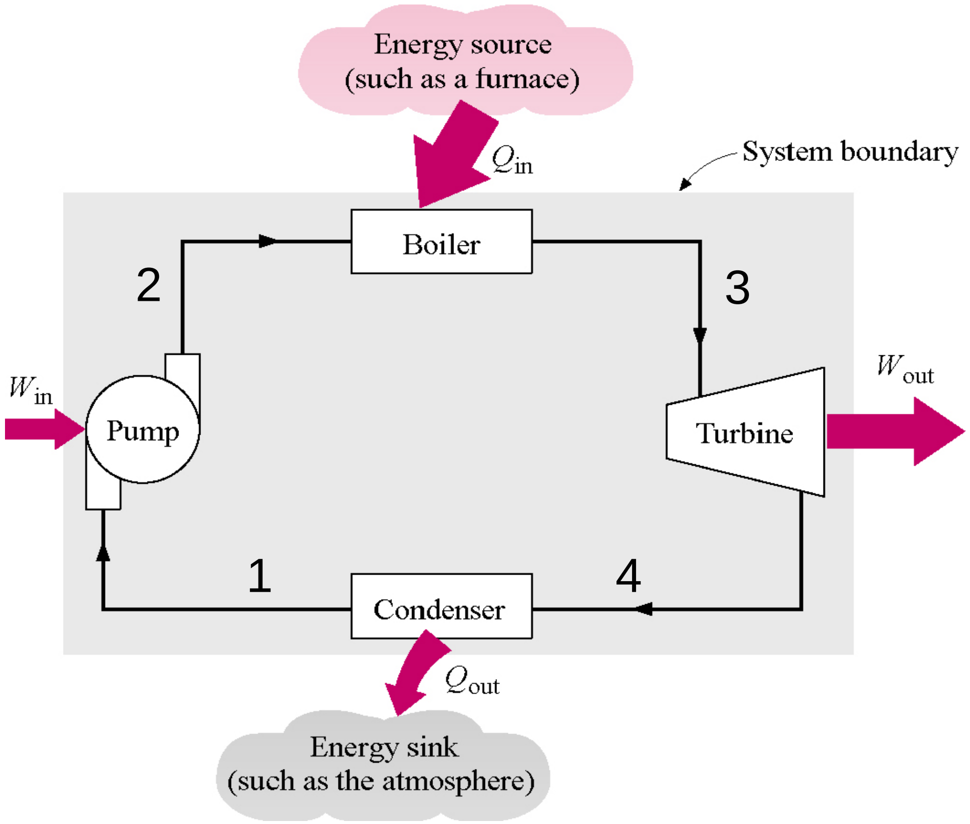
- To increase the thermal efficiency in any power cycle, we try to increase the maximum temperature at which heat is added.

Reasons why the Carnot cycle is not used!

- Pumping process 1-2 requires the pumping of a mixture of saturated liquid and saturated vapor at state 1 and the delivery of a saturated liquid at state 2.

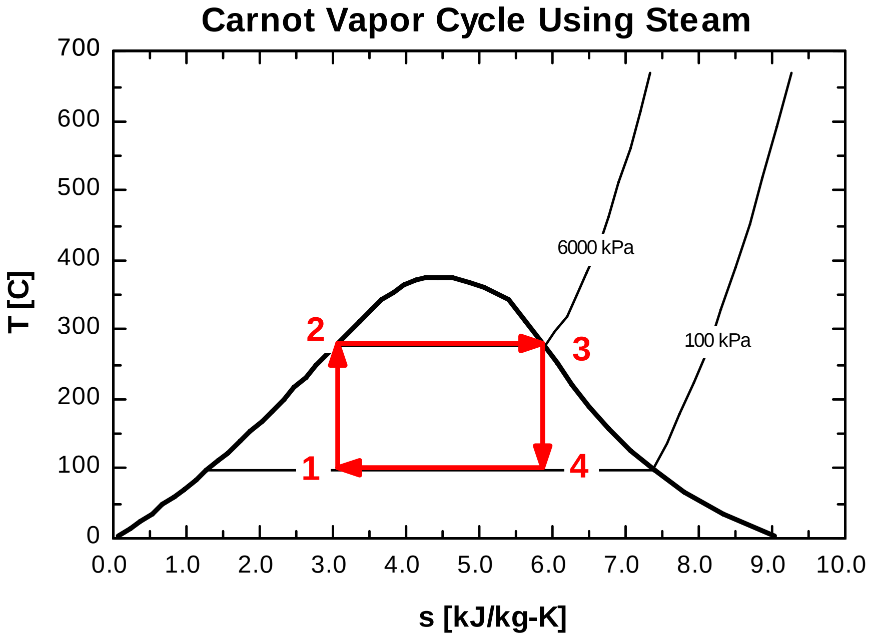
- To superheat the steam to take advantage of a higher temperature, elaborate controls are required to keep T_H constant while the steam expands and does work.

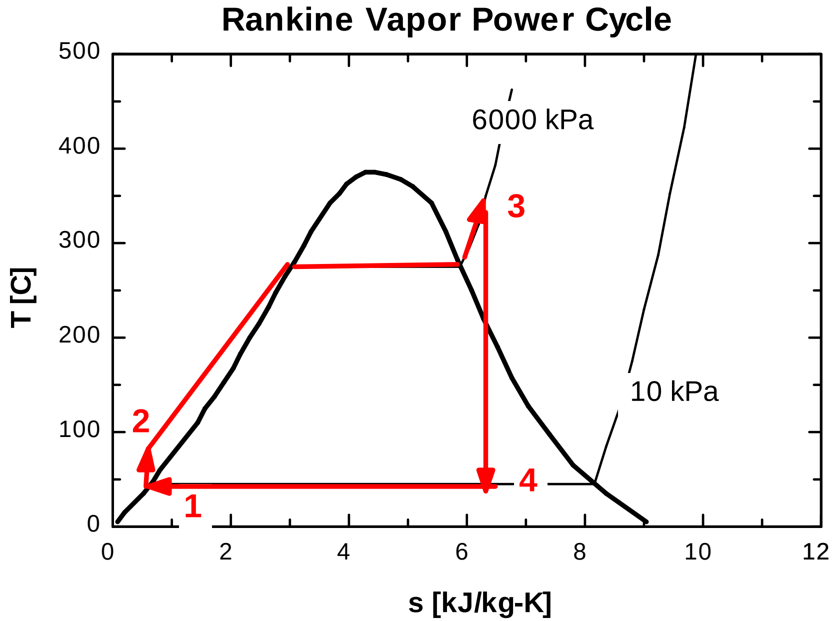




Rankine Cycle

- The simple Rankine cycle continues the condensation process 4-1 until the saturated liquid line is reached.





Ideal Rankine Cycle Processes

Process	Description
1-2	Isentropic compression in pump
2-3	Constant pressure heat addition in boiler
3-4	Isentropic expansion in turbine
4-1	Constant pressure heat rejection in condenser

Example

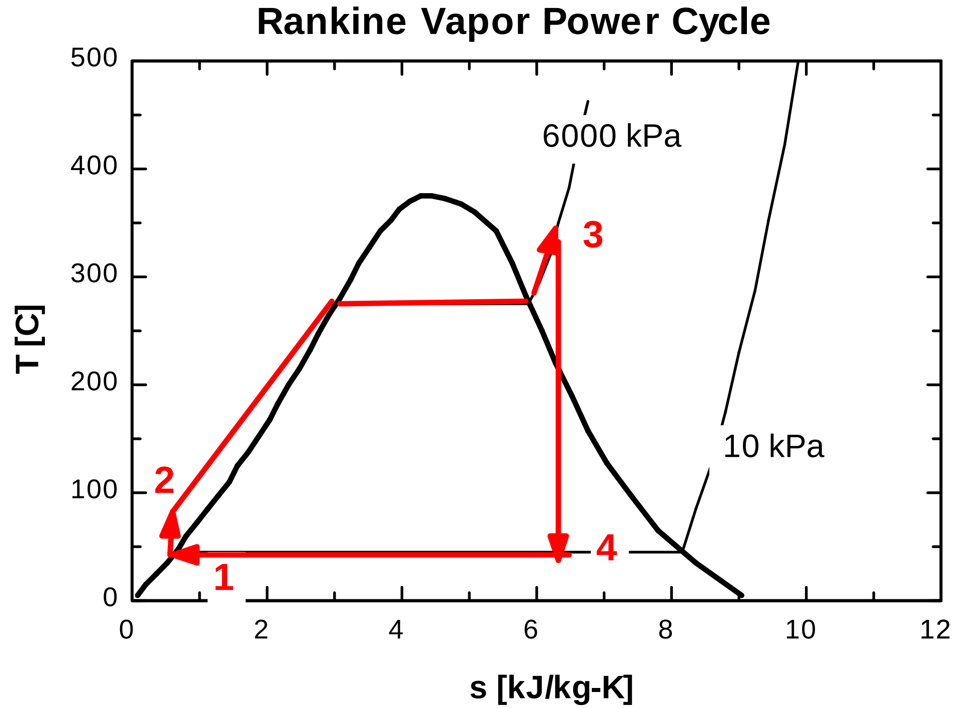
Compute the thermal efficiency of an ideal Rankine cycle for which steam leaves the boiler as superheated vapor at 6 MPa, 350°C, and is condensed at 10 kPa.

Use the power system and *T*-*s* diagram.

$P_2 = P_3 = 6$ MPa = 6000 kPa

$T_3 = 350°$C

$P_1 = P_4 = 10$ kPa



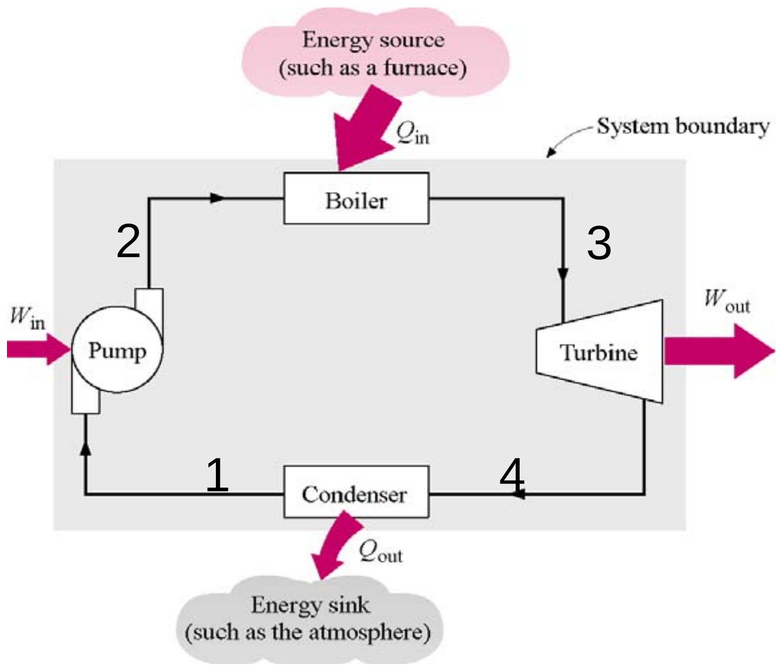
Pump

- Assume steady-flow
- Neglecting ΔPE & ΔKE
- Assume adiabatic and reversible processes

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{m}_1 h_1 + \dot{W}_{pump} = \dot{m}_2 h_2$$

$$\dot{W}_{pump} = \dot{m}(h_2 - h_1)$$



Since the pumping process involves an incompressible liquid, state 2 is in the compressed liquid region

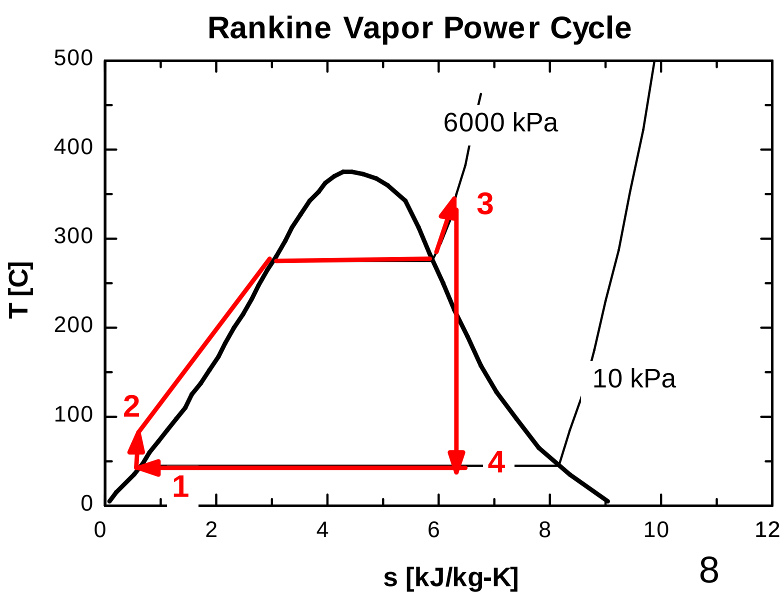
Recall the property relation:

$$dh = T\,ds + v\,dP$$

Since the ideal pumping process 1-2 is isentropic, ds = 0.

$$dh = v\,dP$$

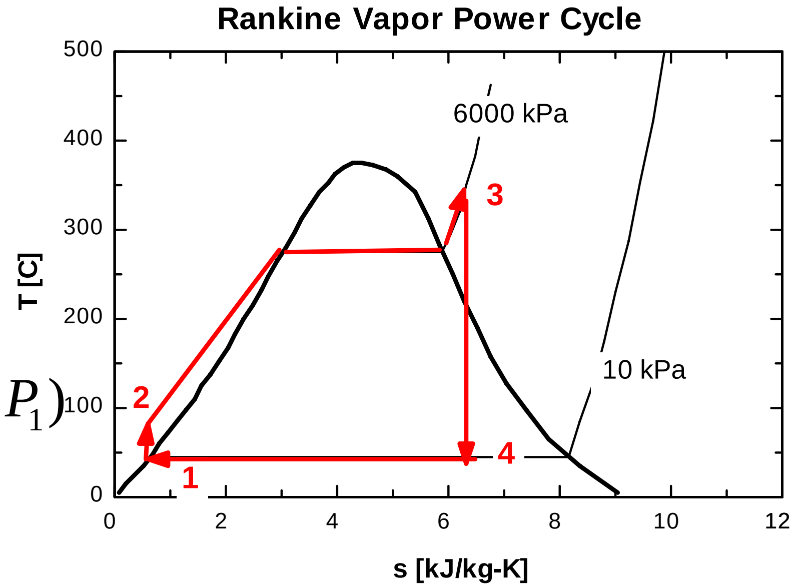
$$\Delta h = h_2 - h_1 = \int_1^2 v\,dP$$



The incompressible liquid assumption allows

$$v \cong v_1 = const.$$

$$h_2 - h_1 \cong v_1 (P_2 - P_1)$$



The pump work :

$$\dot{W}_{pump} = \dot{m}(h_2 - h_1) \cong \dot{m} v_1 (P_2 - P_1)$$

$$w_{pump} = \frac{\dot{W}_{pump}}{\dot{m}} = v_1 (P_2 - P_1)$$

Use the steam tables

$$\left. \begin{array}{l} P_1 = 10 \ kPa \\ Sat.\ liquid \end{array} \right\} \left\{ \begin{array}{l} h_1 = h_f = 191.81 \dfrac{kJ}{kg} \\ v_1 = v_f = 0.00101 \dfrac{m^3}{kg} \end{array} \right.$$

$$w_{pump} = v_1 (P_2 - P_1)$$

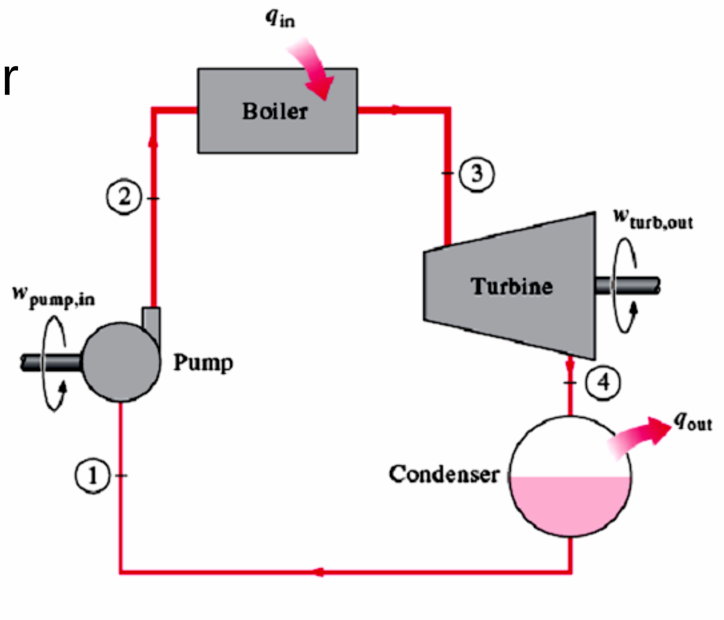
$$= 0.00101 \frac{m^3}{kg} (6000 - 10)\, kPa \frac{kJ}{m^3 kPa} = 6.05 \frac{kJ}{kg}$$

h_2 is found from $$h_2 = w_{pump} + h_1$$

$$= 6.05\frac{kJ}{kg} + 191.81\frac{kJ}{kg} = 197.86\frac{kJ}{kg}$$

Boiler

- To find the heat supplied in the boiler, assume
  - Conservation of mass and energy for steady flow
  - Neglect ΔPE & ΔKE
  - No work is done on the steam in the boiler



$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

$$\dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2)$$

- Find the properties at state 3 from the superheated tables

$$\left.\begin{array}{l} P_3 = 6000\ kPa \\ T_3 = 350^o C \end{array}\right\} \left\{\begin{array}{l} h_3 = 3043.9 \dfrac{kJ}{kg} \\ s_3 = 6.3357 \dfrac{kJ}{kg \cdot K} \end{array}\right.$$

- The heat transfer per unit mass :

$$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = (3043.9 - 197.86)\frac{kJ}{kg} = 2845.1\frac{kJ}{kg}$$

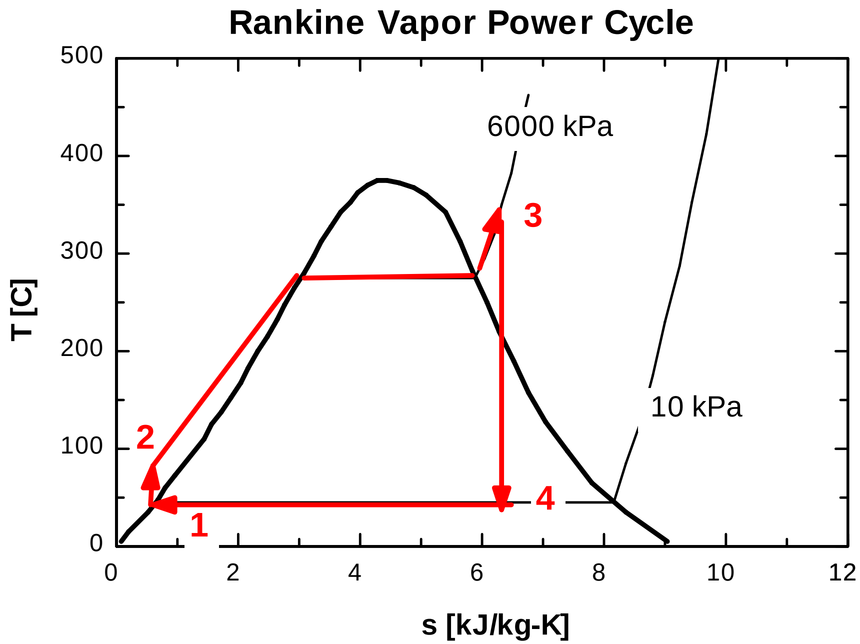
Turbine

- To find turbine work assume
  - Conservation of mass and energy for steady flow.
  - The process is adiabatic and reversible
  - Neglect ΔPE & ΔKE

$$\dot{m}_3 = \dot{m}_4 = \dot{m}$$

$$\dot{m}_3 h_3 = \dot{W}_{turb} + \dot{m}_4 h_4$$

$$\dot{W}_{turb} = \dot{m}(h_3 - h_4)$$



Find the properties at state 4 from the steam tables by noting $s_4 = s_3 = 6.3357$ kJ/kg-K

$$at\ \ P_4 = 10kPa : s_f = 0.6492 \frac{kJ}{kg \cdot K};\ s_g = 8.1488 \frac{kJ}{kg \cdot K}$$

$$s_4 = s_f + x_4 s_{fg}$$

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.3357 - 0.6492}{7.4996} = 0.758$$

$$h_4 = h_f + x_4 h_{fg}$$

$$= 191.81 \frac{kJ}{kg} + 0.758(2392.1) \frac{kJ}{kg}$$

$$= 2005.0 \frac{kJ}{kg}$$

❑ The turbine work per unit mass

$$\boxed{w_{turb} = h_3 - h_4}$$

$$= (3043.9 - 2005.0) \frac{kJ}{kg}$$

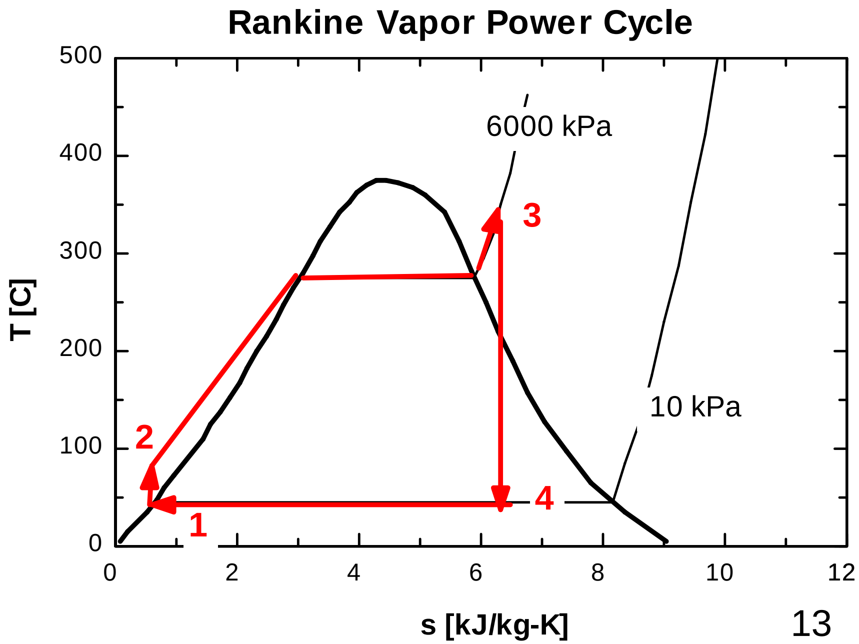
$$= 1038.9 \frac{kJ}{kg}$$

❑ The net work done by the cycle :

$$\boxed{w_{net} = w_{turb} - w_{pump}}$$

$$= (1038.9 - 6.05) \frac{kJ}{kg}$$

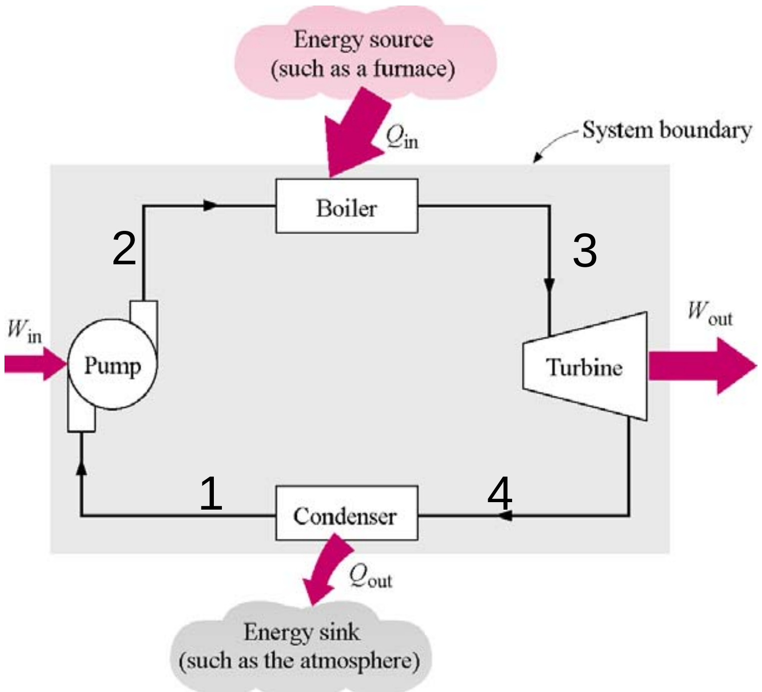
$$= 1032.8 \frac{kJ}{kg}$$

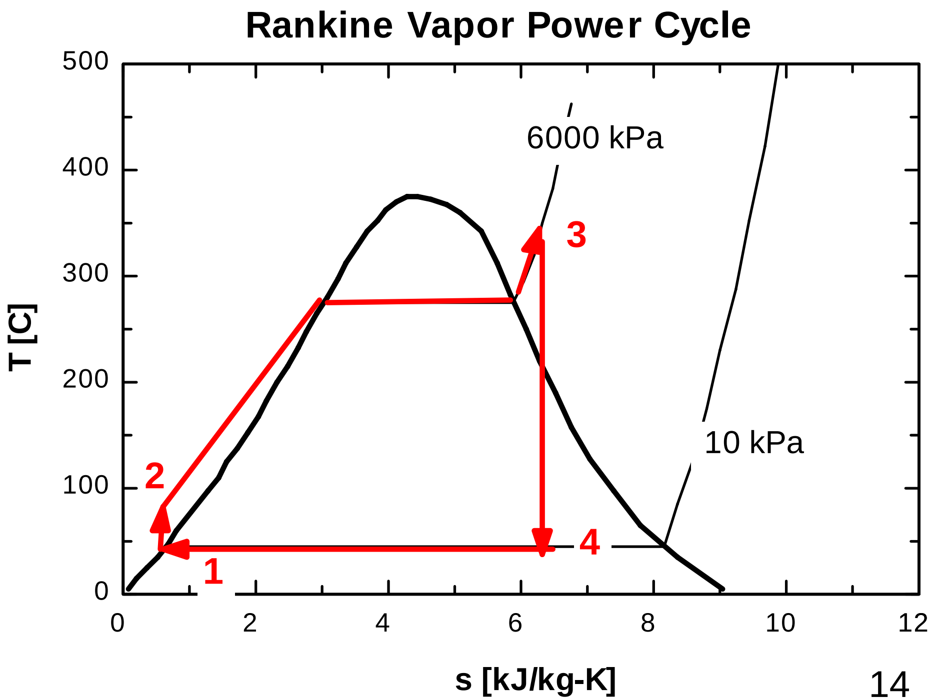


- The thermal efficiency

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{1032.8\,\frac{kJ}{kg}}{2845.1\,\frac{kJ}{kg}}$$

$$= 0.363 \; or \;\; 36.3\%$$





Previous lecture

Ideal Rankine cycle

Analyzing the efficiency of components

Today lecture

Deviation of actual vapor cycles

Way to improve the Rankine cycle

Ideal Rankine cycle with reheat

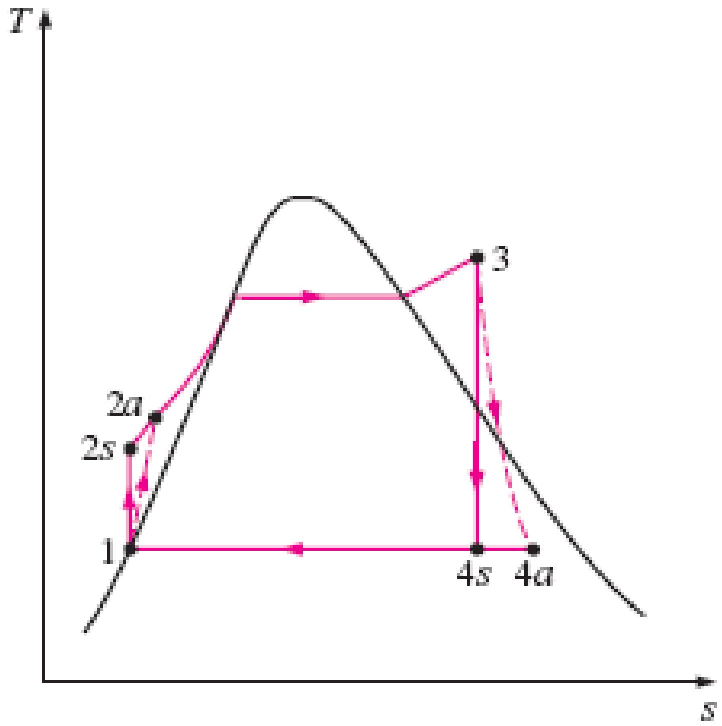
Ideal Rankine cycle with regeneration

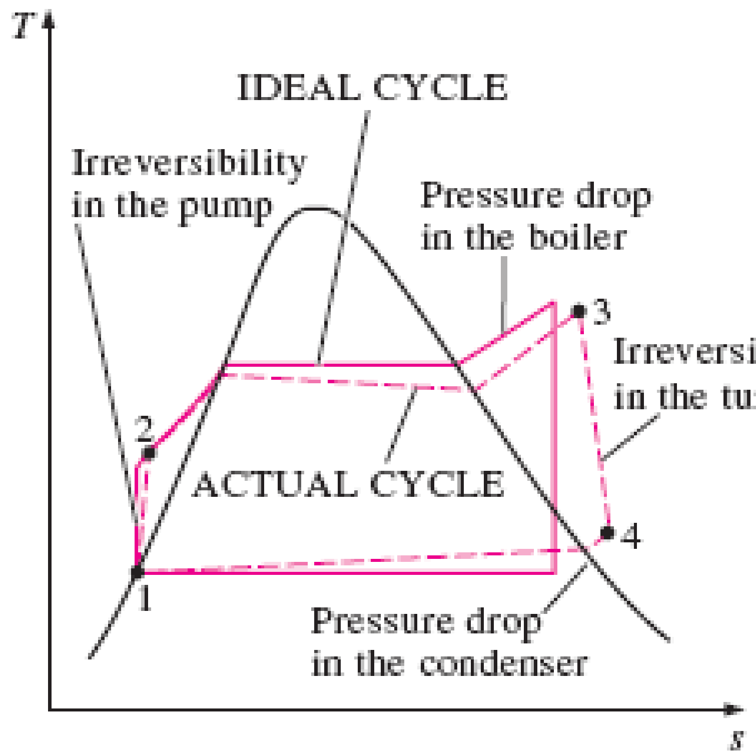
Next lecture

Isentropic efficiency

Cogeneration

Deviation of actual vapor power cycles from idealized cycles





Fluid friction → pressure drops in boiler, condenser, and piping

Irreversibility: heat loss from steam to surroundings

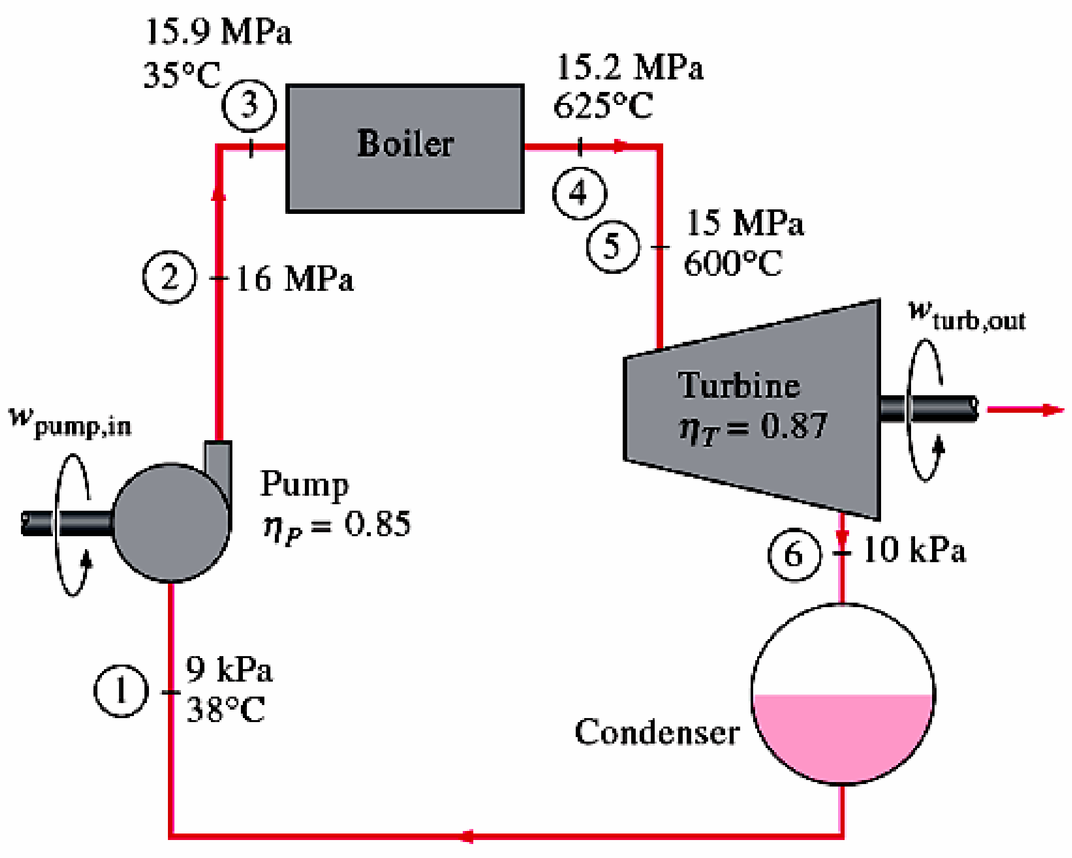
Isentropic efficiency

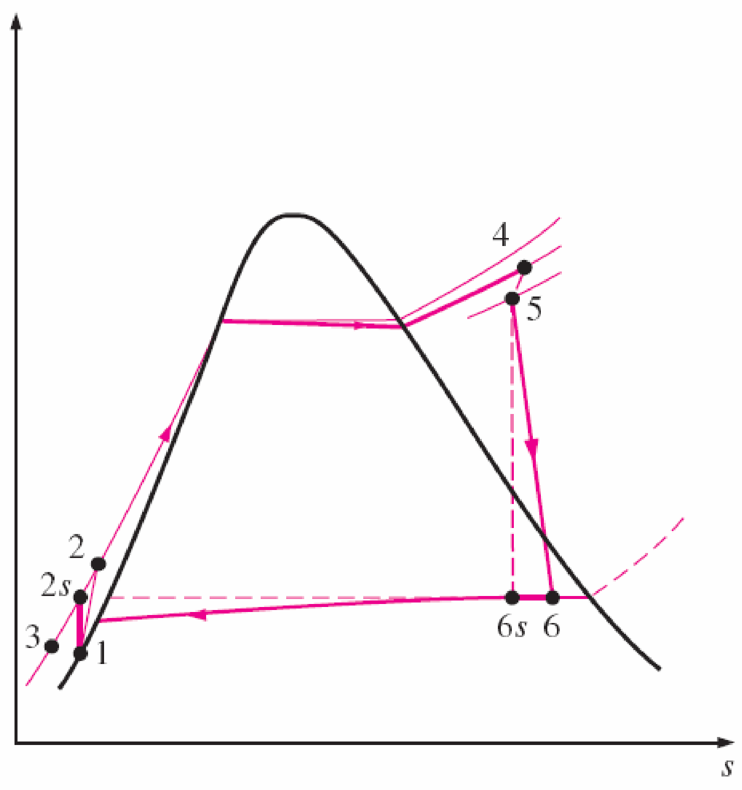
$$\boxed{\eta_P = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1}}$$

$$\boxed{\eta_T = \frac{w_a}{w_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}}$$

Example

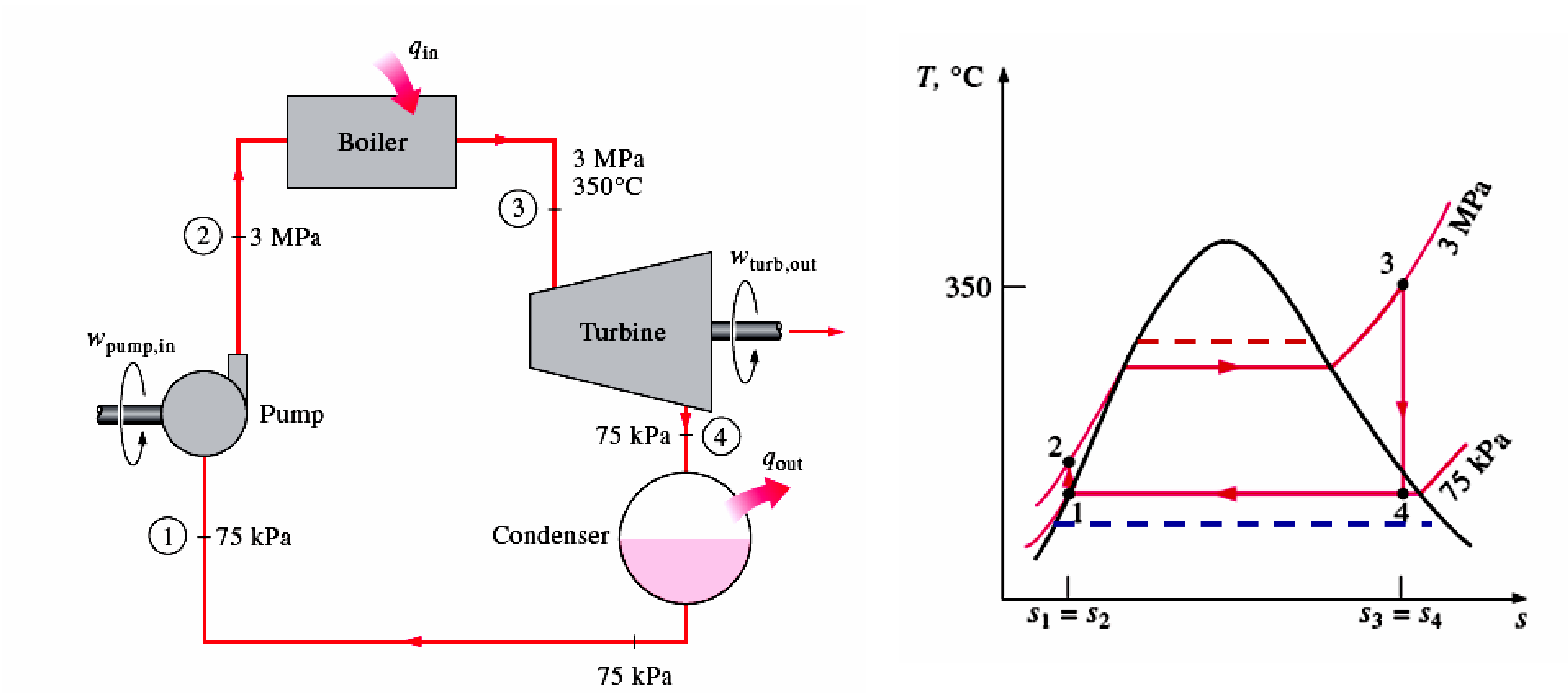
A steam power plant operates on the cycle shown in Fig. If the isentropic efficiency of turbine is 87 percent and the isentropic efficiency of the pump is 85 percent, determine (a) the thermal efficiency of the cycle and (b) the net power output of the plant for a mass flow rate of 15 kg/s





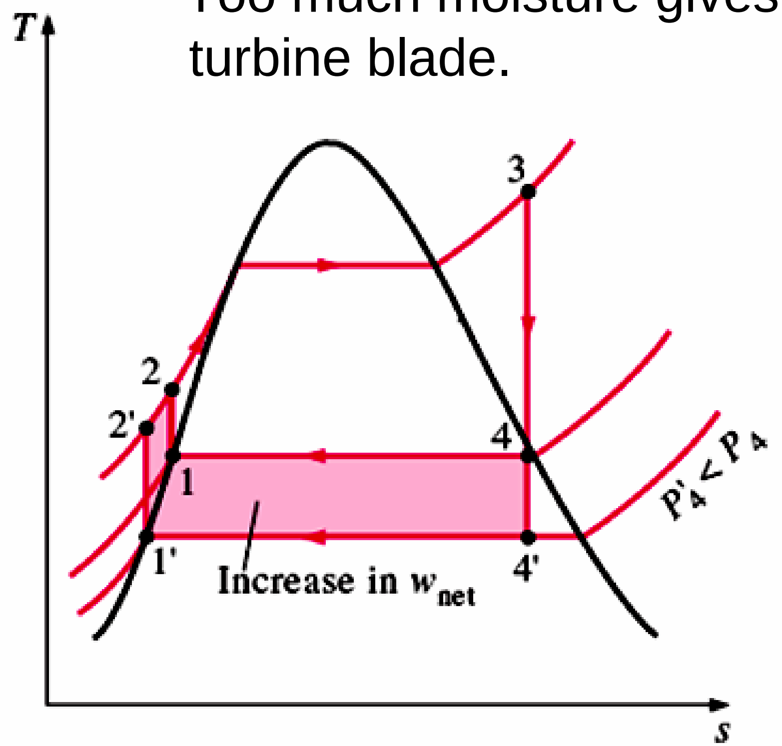
Ways to improve the simple Rankine cycle efficiency

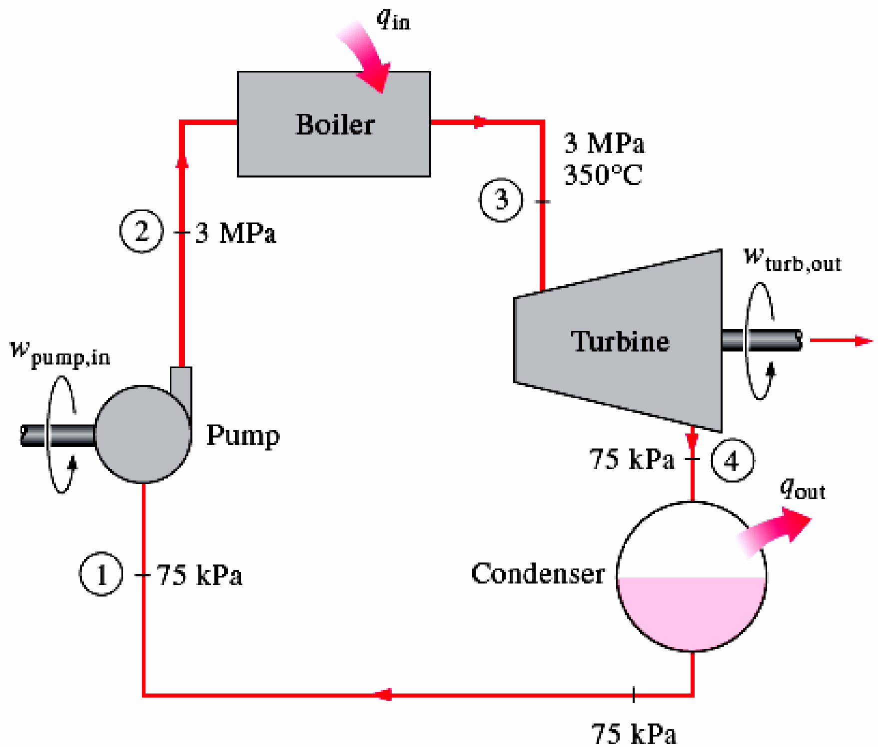
Increase the average temperature at which heat is transferred to the working fluid in the boiler, or decrease the average temperature at which heat is rejected from the working fluid in the condenser.



Ways to improve the simple Rankine cycle efficiency

- **Lower condenser pressure**
  - Less energy is lost to surroundings.
  - Moisture is increased at turbine exit.
  - Min. temp is limited by cooling temperature
  - Lower temperature gives lower pressure, and this might cause the problem of air leakage into condenser.
  - Too much moisture gives lower turbine efficiency and erosion of turbine blade.



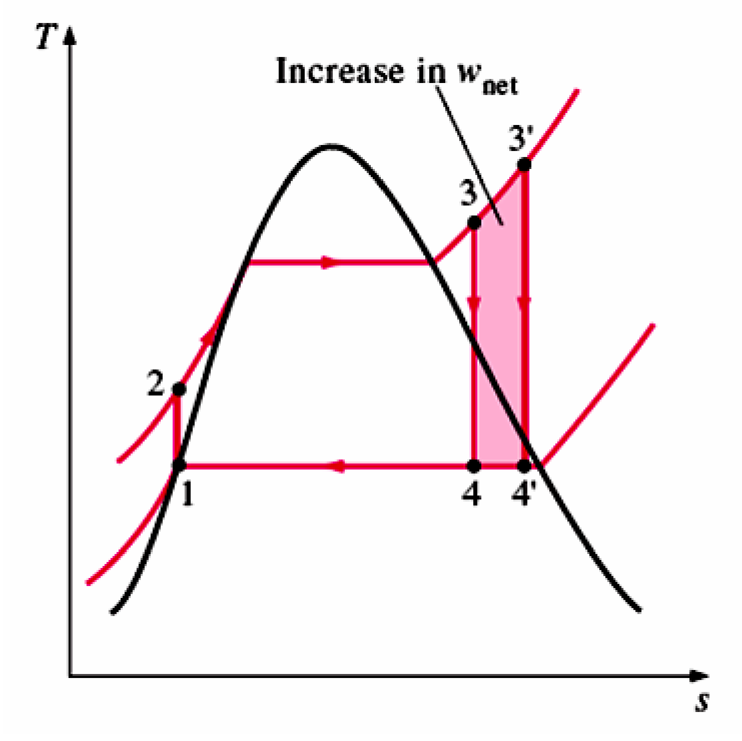


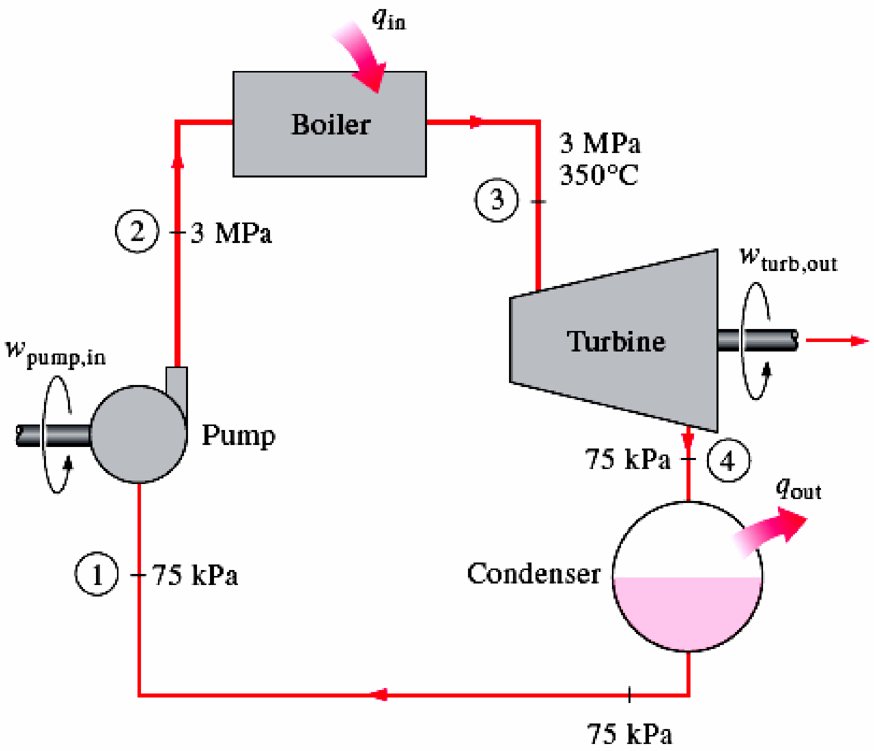
Ways to improve the simple Rankine cycle efficiency

❑ **Superheat the vapor (without increasing the boiler pressure)**

Average temperature is higher during heat addition.

- Moisture is reduced at turbine exit.
- Max. temp is limited by metallurgical property



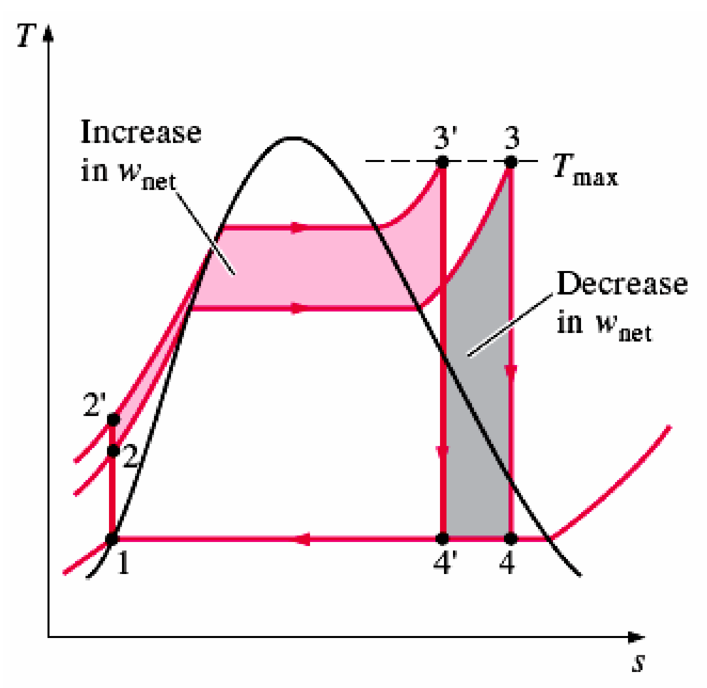


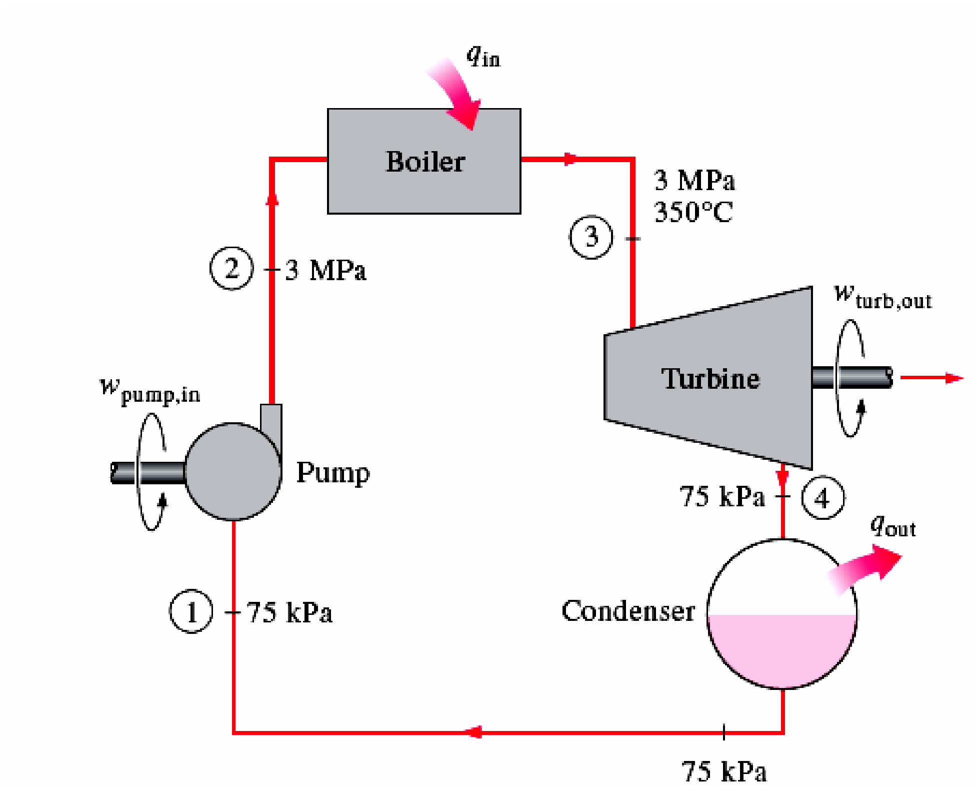
Ways to improve the simple Rankine cycle efficiency

- **Increase boiler pressure (for fixed maximum temperature)**

  - Availability of steam is higher at higher pressures.

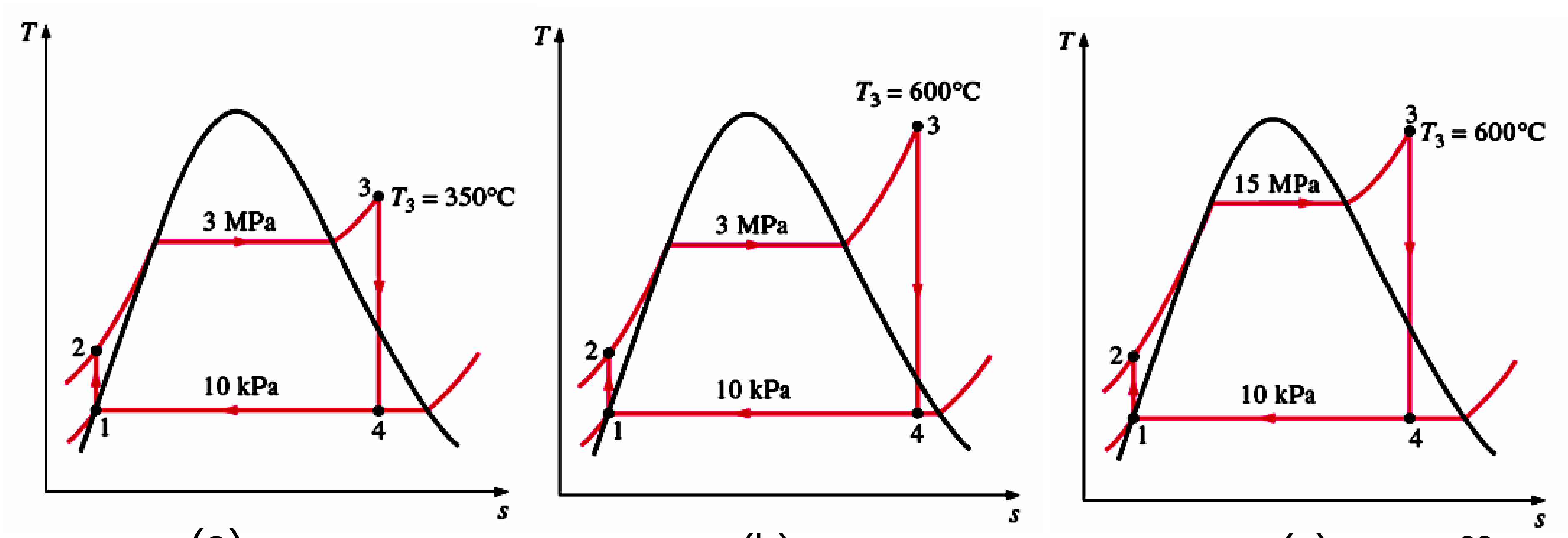
  - Moisture is increased at turbine exit (Side effect and improved by reheating).





Example

Consider a steam power plant operating on the ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350 ^{O}C and is condensed in the condenser at a pressure of 10 kPa. Determine (a) the thermal efficiency of this power plant, (b) the thermal efficiency if steam is superheated to 600^{O}C instead of 350^{O}C, and (c) the thermal efficiency if the boiler pressure is raised to 15 MPa while the turbine inlet temperature is maintained at 600^{O}C.

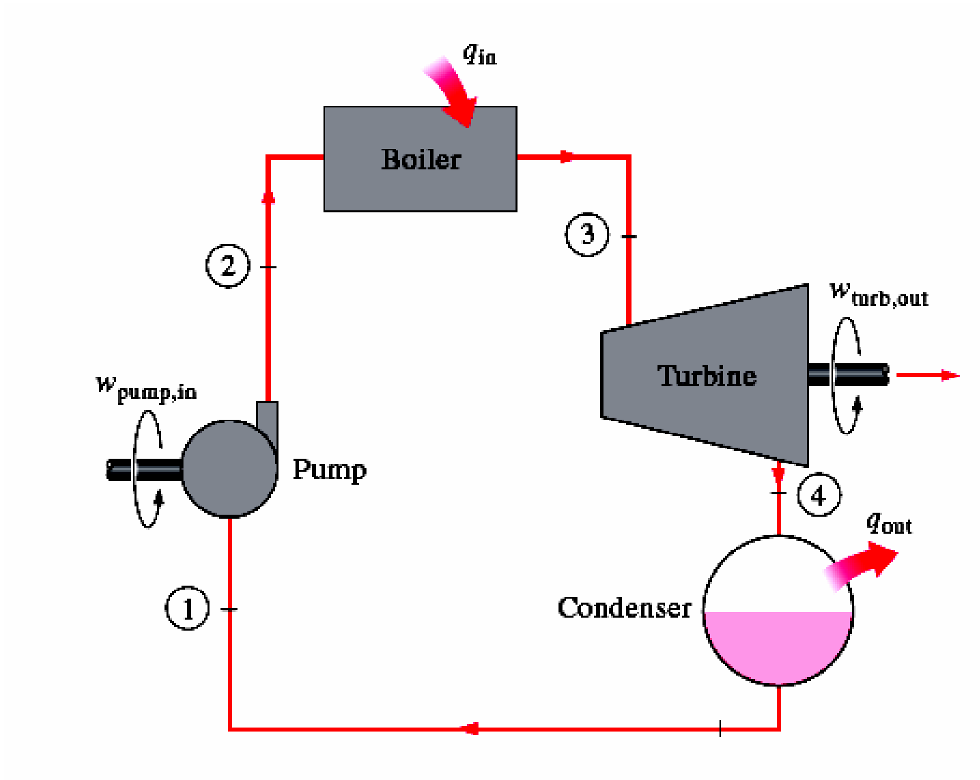


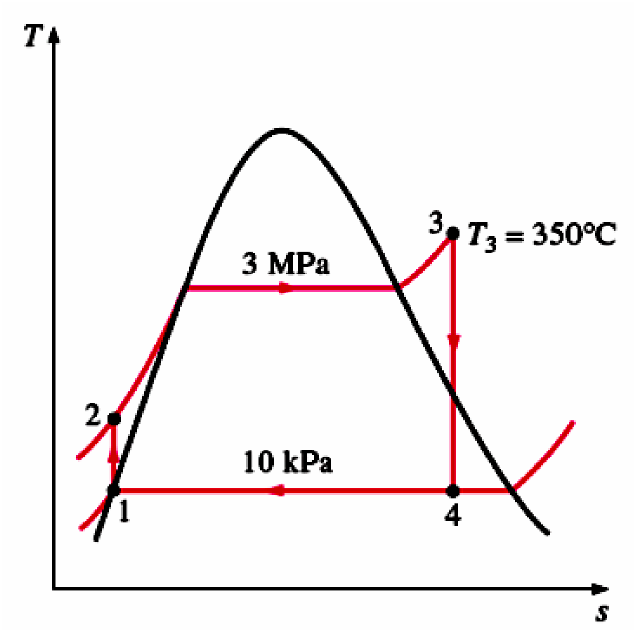
(a) (b) (c)

$$\boxed{\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{in}}{q_{out}}}$$

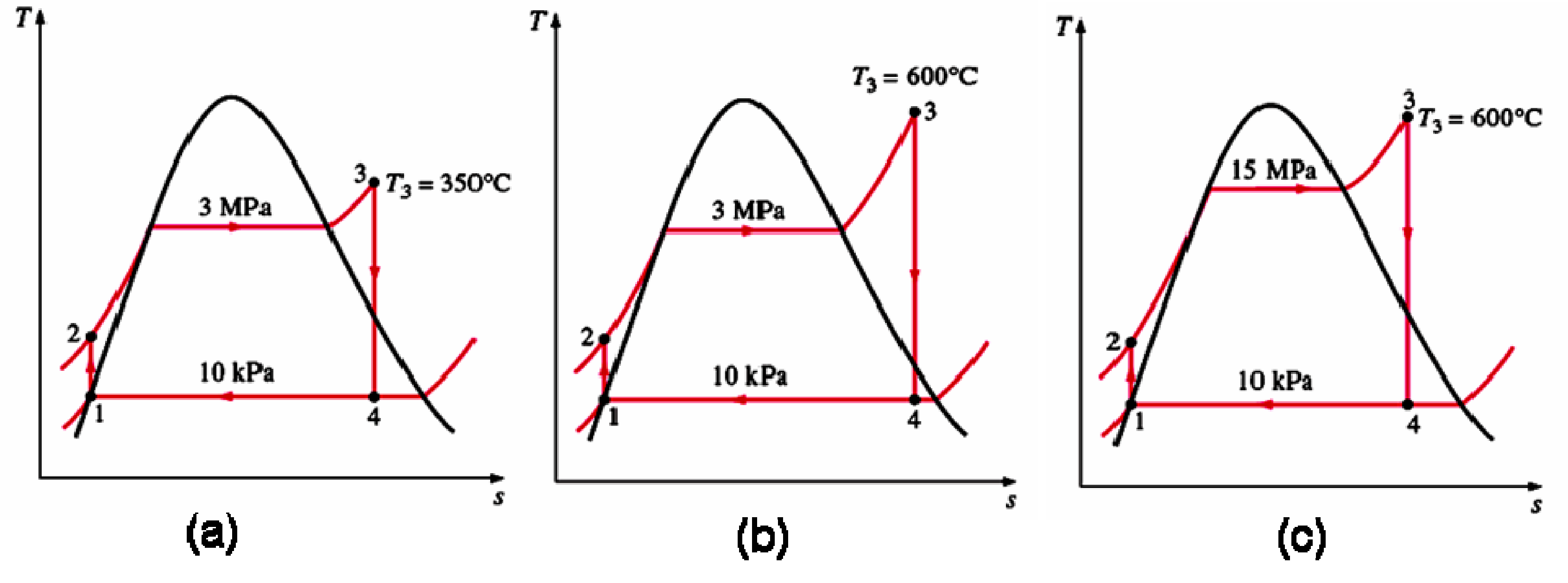
$$q_{in} = h_3 - h_2$$

$$q_{out} = h_4 - h_1$$





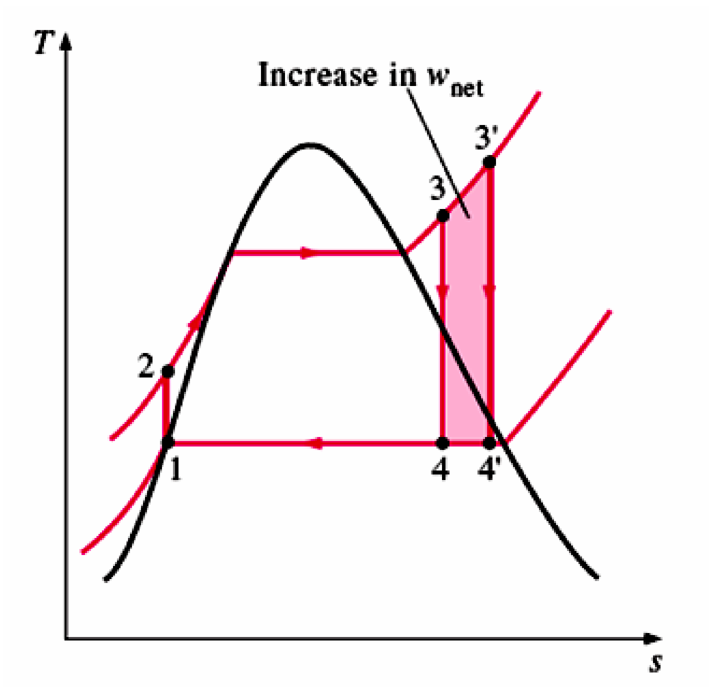
	(a)	(b)	(c)
T_{max}	350 ^oC	600 ^oC	600^oC
P_{max}	3 MPa	3 MPa	15 MPa
x	0.8128	0.915	0.804
q_{in}	2921.3	3488	3376.2
q_{out}	1944.3	2188.5	1923.5
η_{th}	33.4%	37.3%	43%

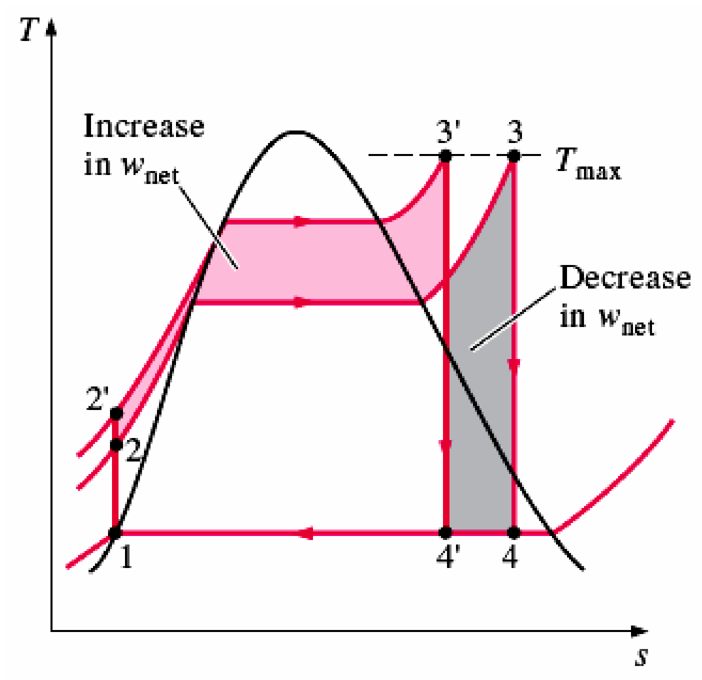


Ideal Reheat Rankine Cycle

As the boiler pressure is increased in the simple Rankine cycle, not only does the thermal efficiency increase, but also the turbine exit moisture increases.

1. Superheat the steam to very high temperature before it enters the turbine (limited by metallurgical property)
2. Expand the steam in the turbine in two states, and reheat it in between.

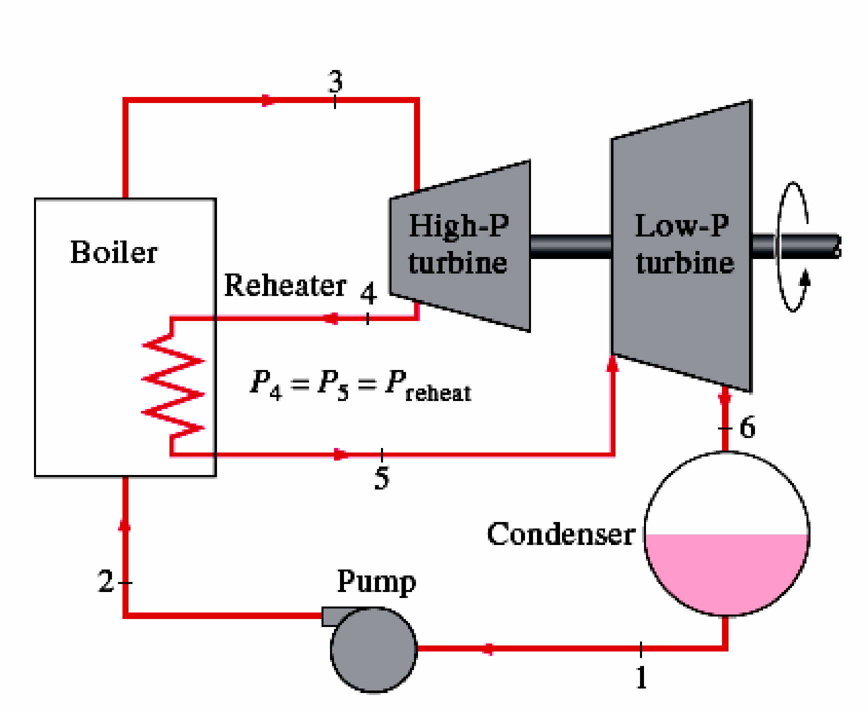


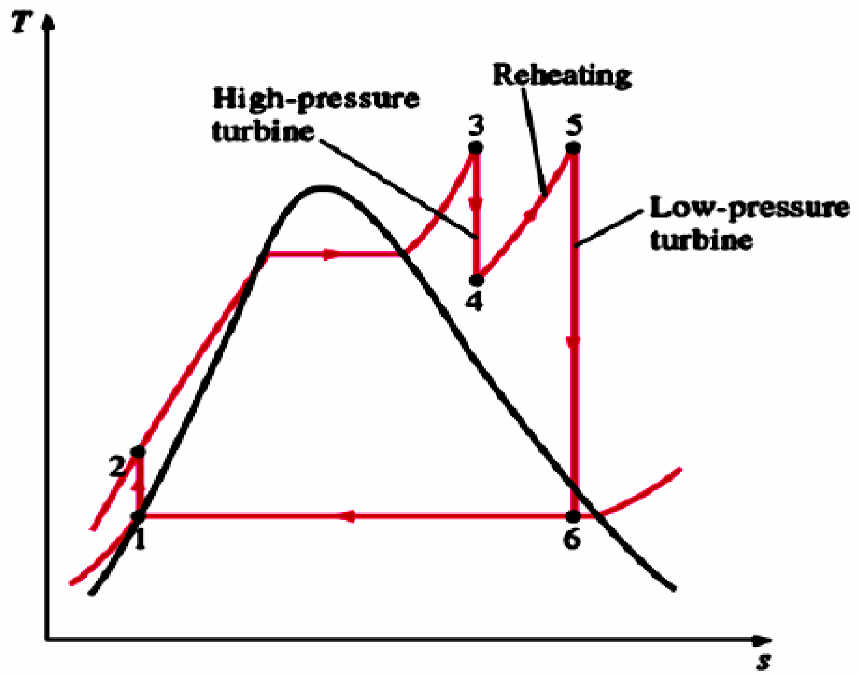


Ideal Reheat Rankine Cycle

$$w_{turb} = w_{turb,1} + w_{turb,2} = (h_3 - h_4) + (h_5 - h_6)$$

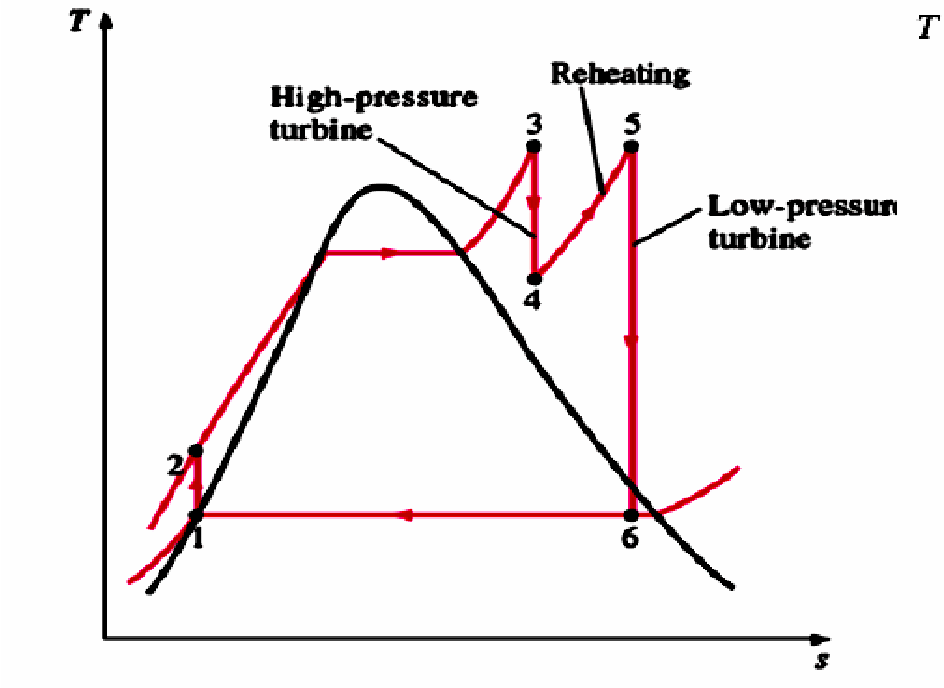
$$q_{in} = q_{primary} + q_{reheat} = (h_3 - h_2) + (h_5 - h_4)$$

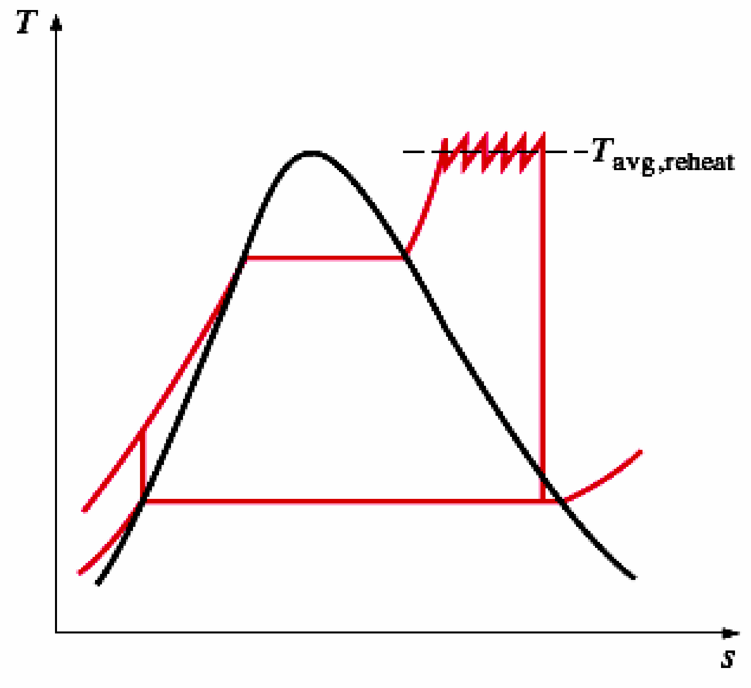




Ideal Reheat Rankine Cycle

- The average temperature during the reheat process can be increased by increasing the number of expansion and reheat stages.
- As the number of stages is increased, the expansion and reheat processes approach an isothermal process at the max. temp.
- In theory, efficiency from the second reheat is about half of that from a single reheat. And if the turbine inlet pressure is not high enough, double reheat would result in superheated exhaust (increase the average temperature for heat rejection) → super critical pressure P > 22.06 MPa





Rankine Cycle with Reheat

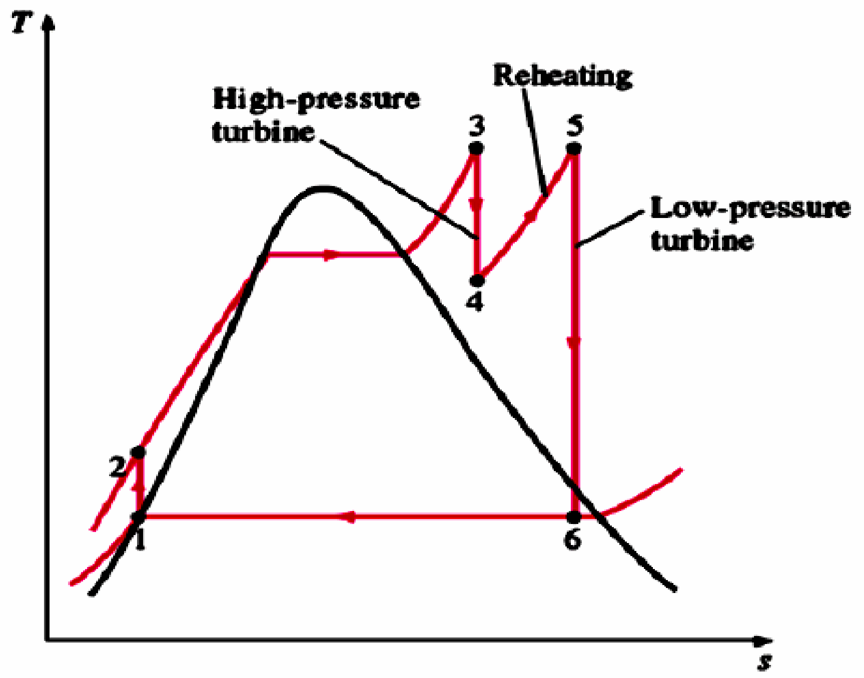
Component	Process	First Law Result	
Boiler	Const. *P*	q_{in}	$= (h_3 - h_2) + (h_5 - h_4)$
Turbine	Isentropic	w_{out}	$= (h_3 - h_4) + (h_5 - h_6)$
Condenser	Const. *P*	q_{out}	$= (h_6 - h_1)$
Pump	Isentropic	w_{in}	$= (h_2 - h_1) = v_1(P_2 - P_1)$

The thermal efficiency is given by

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

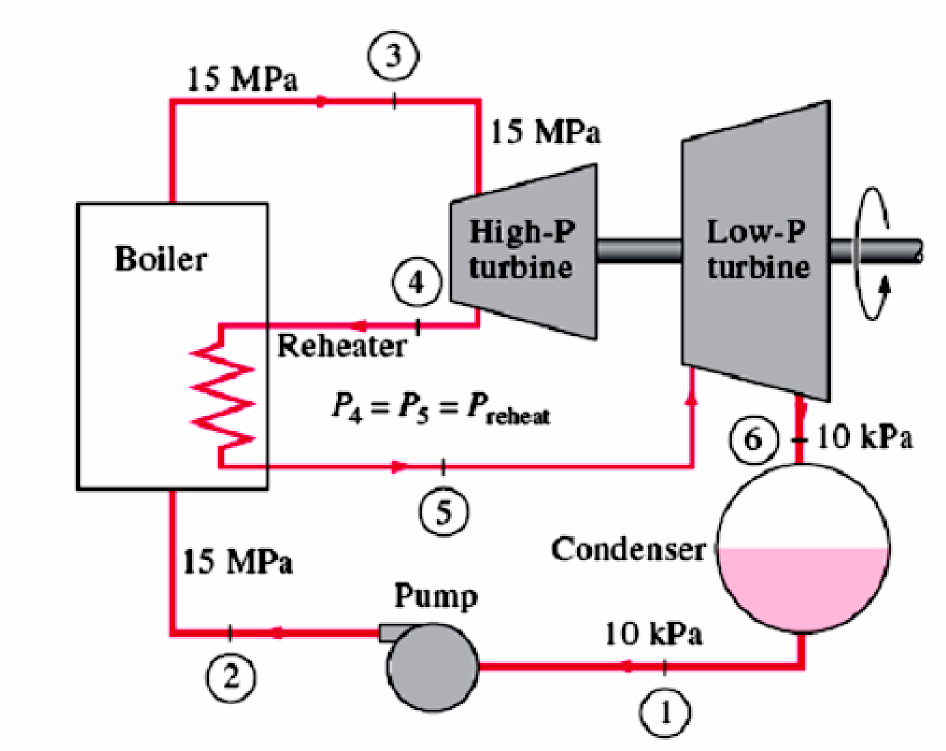
$$= \frac{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)}{(h_3 - h_2) + (h_5 - h_4)}$$

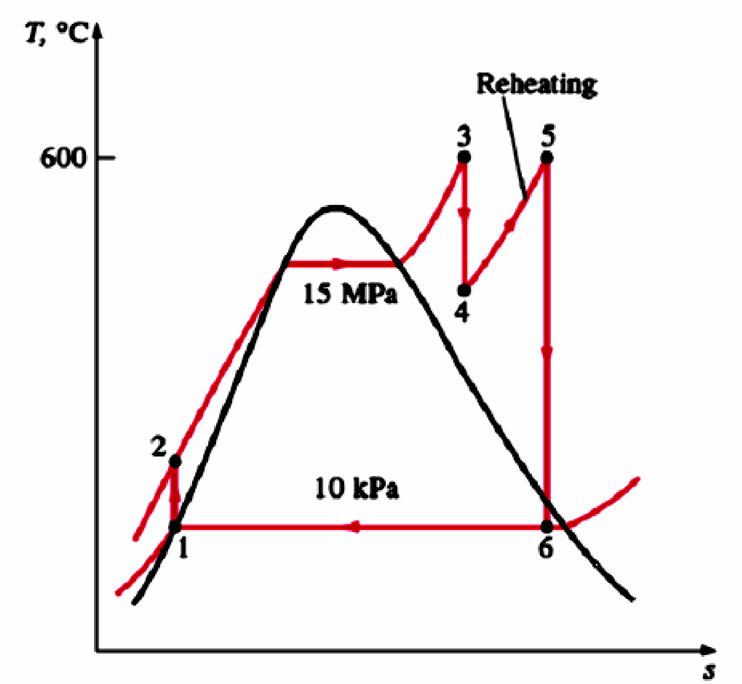
$$= 1 - \frac{h_6 - h_1}{(h_3 - h_2) + (h_5 - h_4)}$$



Example

Consider a steam power plant operating on the ideal reheat Rankine cycle. Steam enters the high-pressure turbine at 15 MPa and 600^{O}C and is condensed in the condenser at a pressure of 10 kPa. If the moisture content of the steam at the exit of the low-pressure turbine is not to exceed 10.4 percent, determine (a) the pressure at which the steam should be reheated and (b) the thermal efficiency of the cycle. Assume the steam is reheated to the inlet temperature of the high-pressure turbine.





Assumptions

1. Steady operating conditions
2. ΔKE & ΔPE are negligible
3. Ideal reheat Rankine cycle

(a) Pressure at which the steam should be reheated ($P_5 = ?$)

$T_5 = 600°C$

$s_5 = s_6$

State 6: $P_6 = 10 \text{ kPa}$

$x_6 = 0.896$ (mixture)

$$s_6 = s_f + x_6 s_{fg}$$
$$= 0.6492 + 0.896\,(7.4996)$$
$$= 7.3688 \text{ kJ/kg.K.}$$

$$h_6 = h_f + x_6 h_{fg}$$

$$= 191.81 + 0.896(2392.1)$$

$$= 2335.1 \ \text{kJ/kg}$$

Thus. state 5: $\left.\begin{array}{l} T_5 = 600\,^\circ\text{C} \\ s_5 = s_6 \end{array}\right\}$ $\begin{array}{l} P_5 = 4.0 \ \text{MPa} \\ h_5 = 3674.9 \ \dfrac{\text{kJ}}{\text{kg}} \end{array}$

Therefore, steam should be reheated at 4 MPa or. lower to prevent a moisture content above 10.4 %

(b) Thermal efficiency, $\eta_{th} = 1 - \frac{q_{in}}{q_{out}}$

$$q_{in} = (h_3 - h_2) + (h_5 - h_4)$$

$$q_{out} = h_6 - h_1$$

State 1: $P_1 = 10$ kPa, Sat. liquid. $\Big\}$ $h_1 = h_{f@10kPa} = 191.81 \ \frac{kJ}{kg}$

$v_1 = v_{f@10kPa} = 0.00101 \ \frac{m^3}{kg}$

State 2: $P_2 = 15$ MPa

$s_2 = s_1$

$$w_{pump,in} = v_1 (P_2 - P_1) =$$

$$= (0.00101 \ \frac{m^3}{kg})((15{,}000 - 10)\ kPa)\left(\frac{1\ kJ}{1\ kPa.m^3}\right)$$

$$= 15.14 \ kJ/kg$$

$$h_2 = h_1 + w_{pump,in}$$
$$= (191.81 + 15.14)$$
$$= 206.95 \ \frac{kJ}{kg}$$

State 3: $\left.\begin{array}{l} P_3 = 15\ MPa \\ T_3 = 600\ ^\circ C \end{array}\right\}$ $\begin{array}{l} h_3 = 3583.1\ kJ/kg \\ s_3 = 6.6796\ \frac{kJ}{kg.K} \end{array}$

State 4: $\left.\begin{array}{l} P_4 = 4\ MPa \\ s_4 = s_3 \end{array}\right\}$ $\begin{array}{l} h_4 = 3155\ kJ/kg \\ (T_4 = 375.5\ ^\circ C) \end{array}$

Thus

$$q_{in} = (h_3 - h_2) + (h_5 - h_4)$$
$$= (3583.1 - 206.95) + (3674.9 - 3155)$$
$$= 3896.1 \ \frac{kJ}{kg}$$

$$q_{out} = h_6 - h_1$$

$$= 2335.1 - 191.85$$

$$= 2143.3 \quad \frac{kJ}{kg}$$

$$\eta_{th} = 1 - \frac{q_{in}}{q_{out}}$$

$$= 1 - \frac{2143.3}{3896.1} = 0.45 \text{ or } 45\%$$

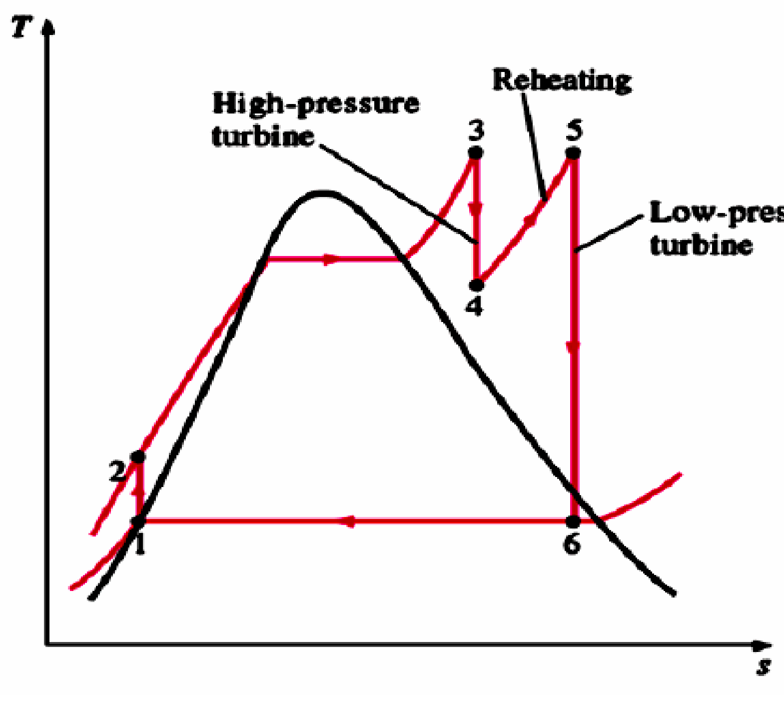
Compare with. ideal Rankine cycle without reheating,

	x	η_{th}
No. reheat	0.804	43%
With reheat	0.896	45%

Assignment 5

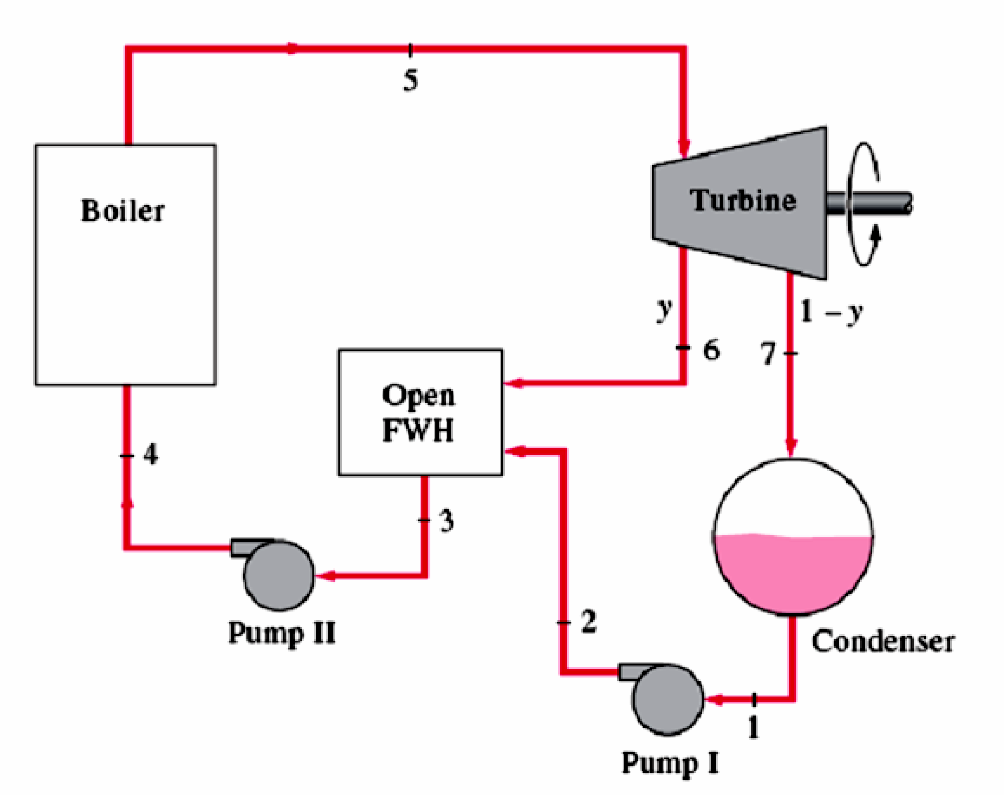
Compare the thermal efficiency and turbine-exit quality at the condenser pressure for a simple Rankine cycle and the reheat cycle when the boiler pressure is 4 MPa, the boiler exit temperature is 400°C, and the condenser pressure is 10 kPa. The reheat takes place at 0.4 MPa and the steam leaves the reheater at 400°C.

	η_{th}	$x_{turb\ exit}$
No Reheat	35.3%	0.8159
With Reheat	35.9%	0.9664



Ideal Regenerative Rankine Cycle

- To improve the cycle thermal efficiency, the average temperature at which heat is added must be increased.
- Allow the steam leaving the boiler to expand the steam in the turbine to an intermediate pressure.

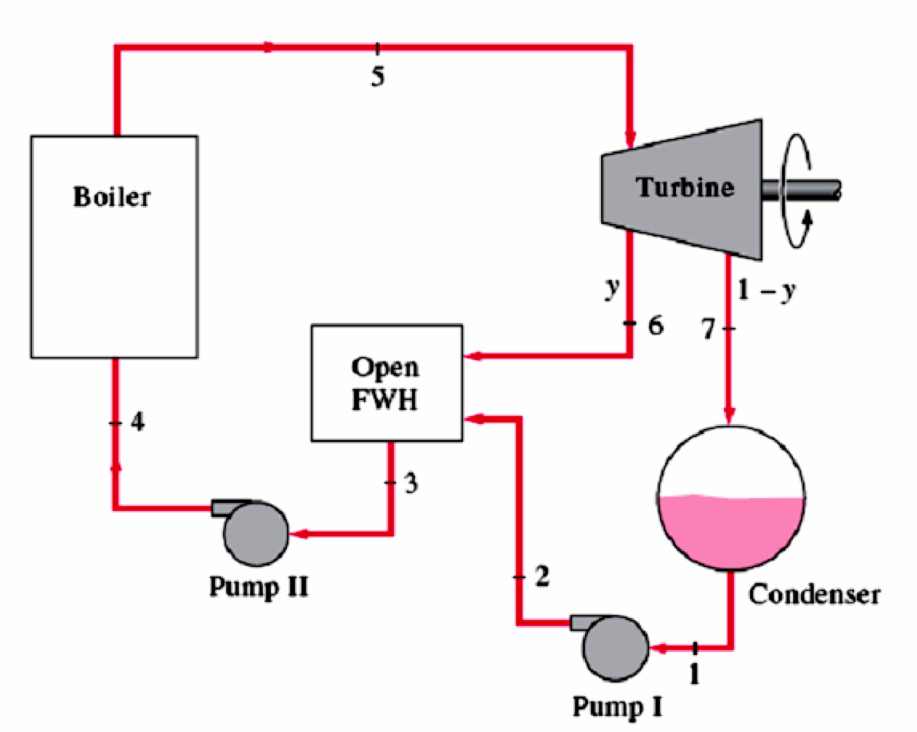


- Some of steam from turbine is sent to a regenerative heater to preheat the condensate before entering the boiler.→ increase feeding water (boiler) temp.

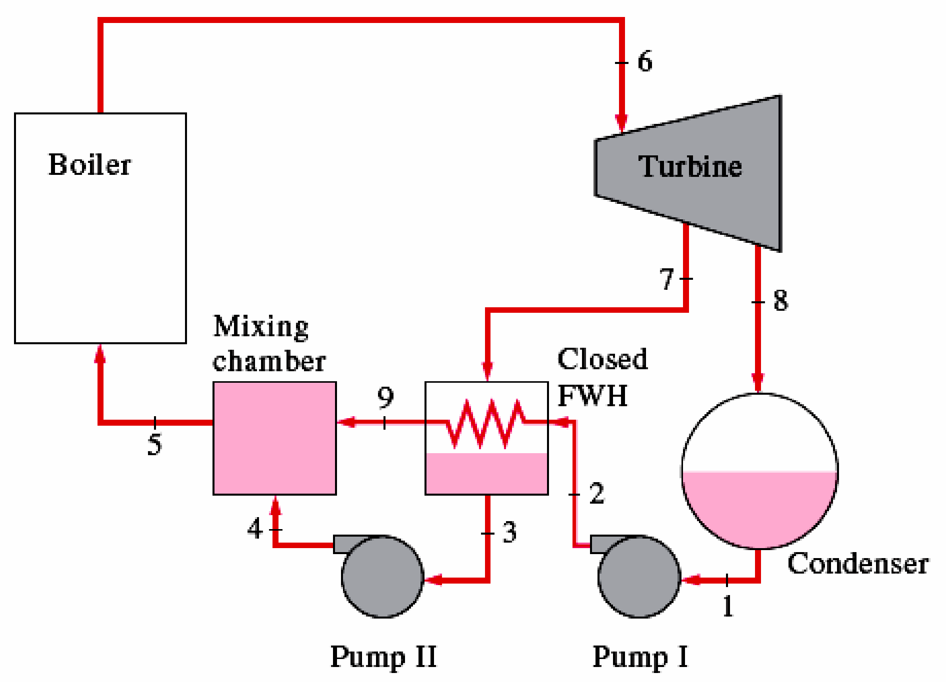
- However, this reduces the mass of steam expanding in the lower-pressure stages of the turbine, and, thus, the total work done by the turbine. The work that is done is done more efficiently.

The preheating of the condensate is done in a combination of open and closed heaters.

- In the open feed water heater, the extracted steam and the condensate are physically mixed.
- In the closed feed water heater, the extracted steam and the condensate are not mixed.



Open feed water heater



Closed feed water heater

Cycle with an open feedwater heater

$$q_{in} = h_5 - h_4$$

$$q_{out} = (1 - y)(h_7 - h_1)$$

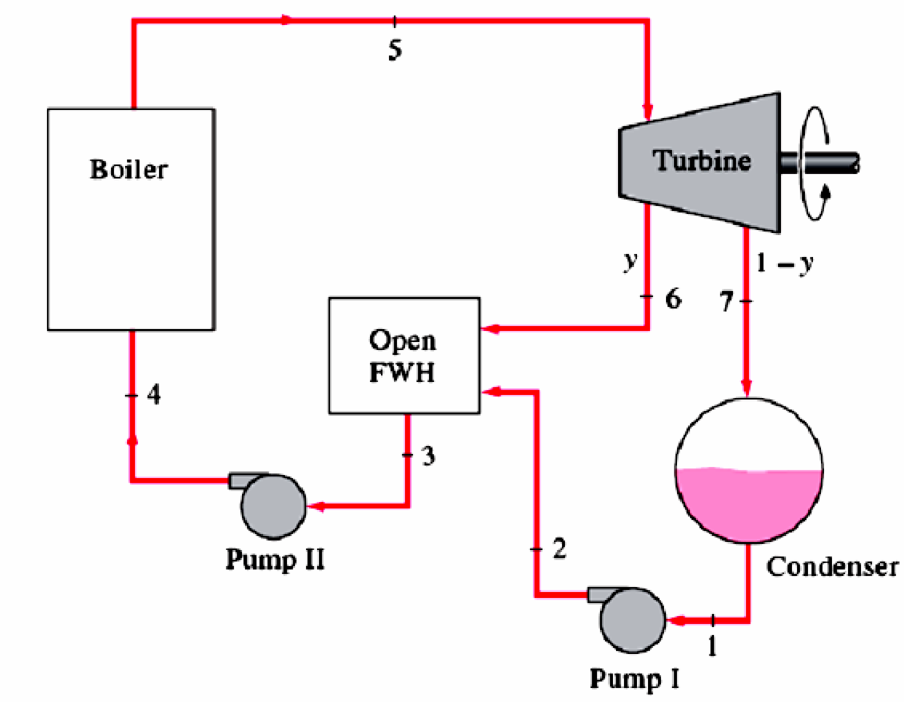
$$w_{turb,out} = (h_5 - h_6) + (1 - y)(h_6 - h_7)$$

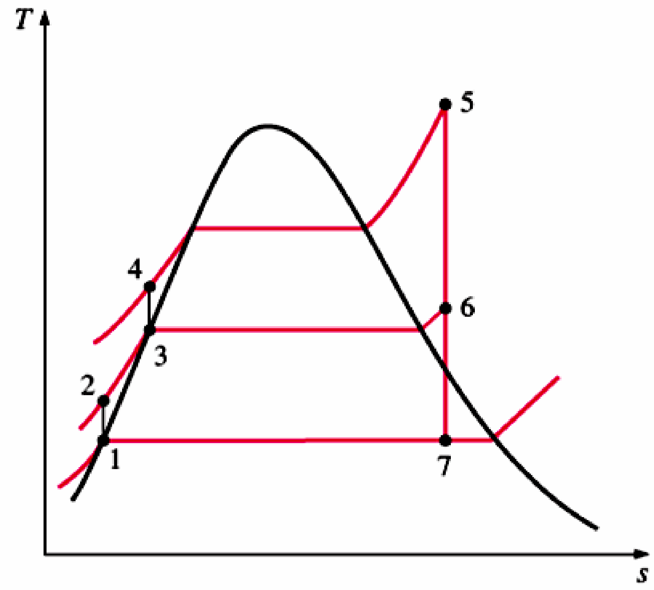
$$w_{pump,in} = (1 - y)\, w_{pump,1,in} + w_{pump,2,in}$$

$$y = \dot{m}_6 / \dot{m}_5$$

$$w_{pump,1,in} = v_1 (P_2 - P_1)$$

$$w_{pump,2,in} = v_3 (P_4 - P_3)$$





Let $y = \dot{m}_6 / \dot{m}_5$ be the fraction of mass extracted from the turbine for the feedwater heater.

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_6 + \dot{m}_2 = \dot{m}_3 = \dot{m}_5$$

$$\dot{m}_2 = \dot{m}_5 - \dot{m}_6 = \dot{m}_5(1 - y)$$

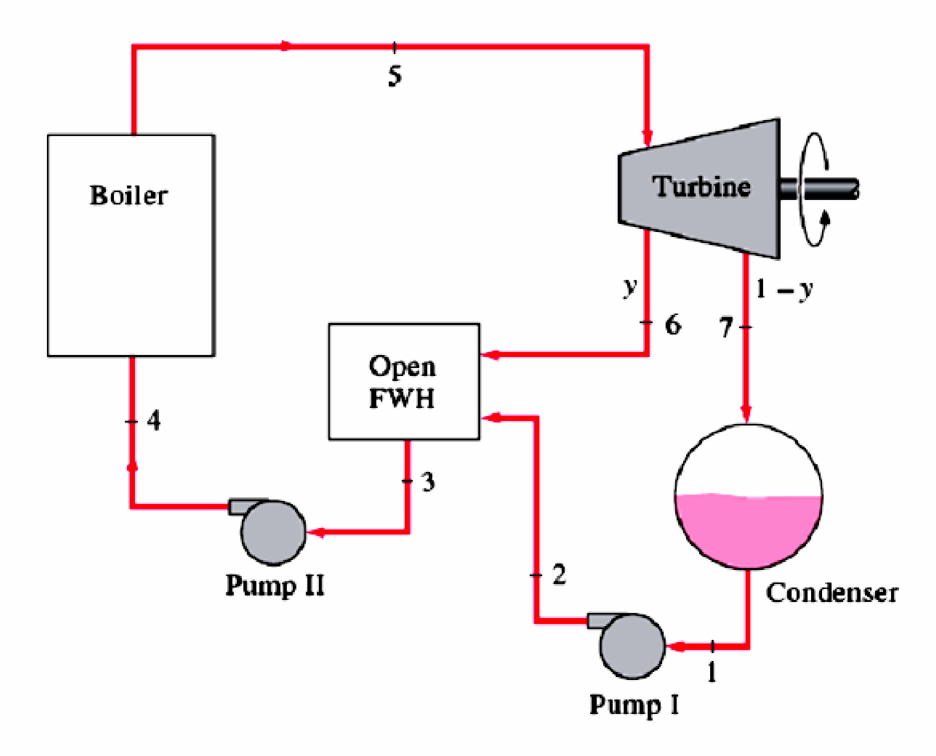
Conservation of energy for the open feedwater heater:

$$\dot{E}_{in} = \dot{E}_{out}$$

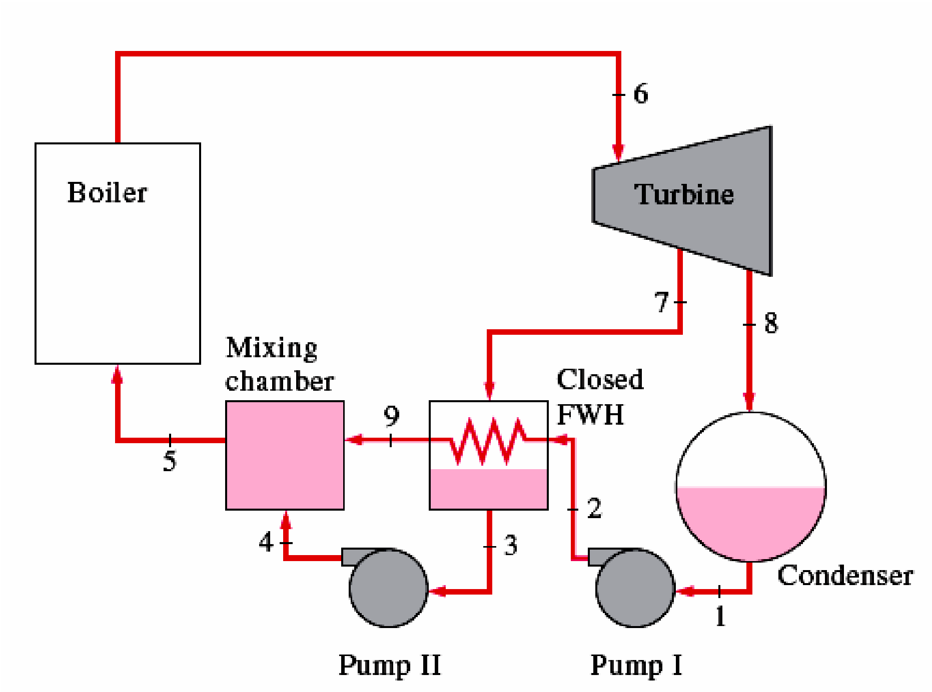
$$\dot{m}_6 h_6 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

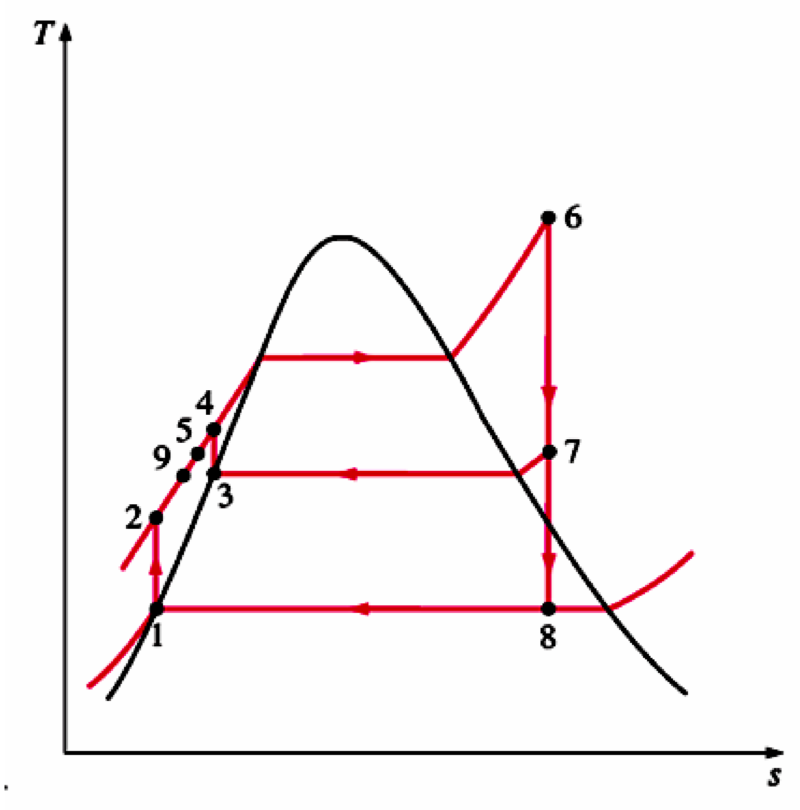
$$y\dot{m}_5 h_6 + (1 - y)\dot{m}_5 h_2 = \dot{m}_5 h_3$$

$$y = \frac{h_3 - h_2}{h_6 - h_2}$$



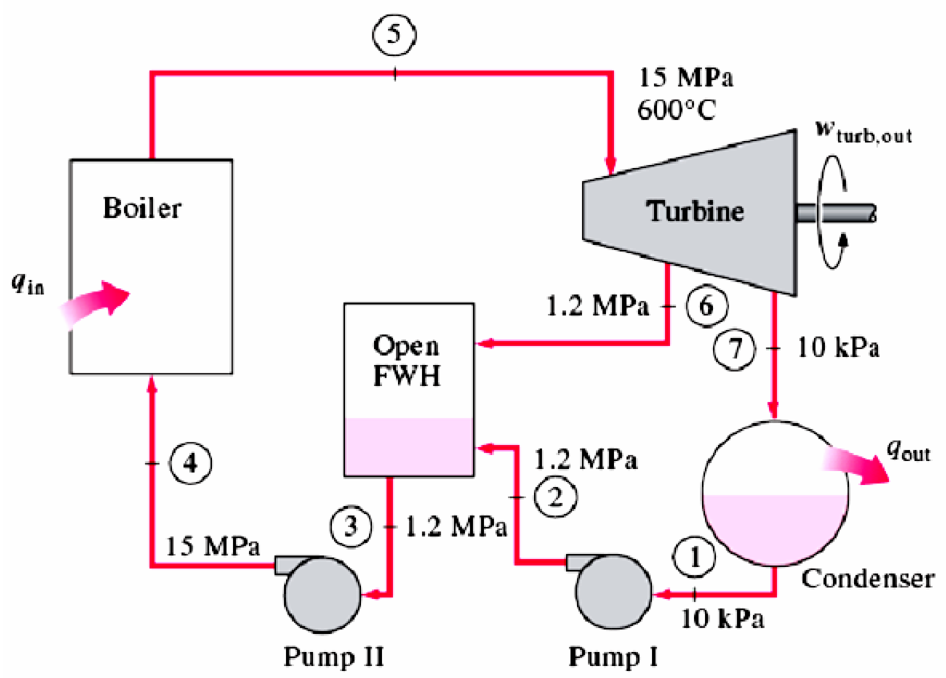
Cycle with a closed feedwater heater with pump to boiler pressure

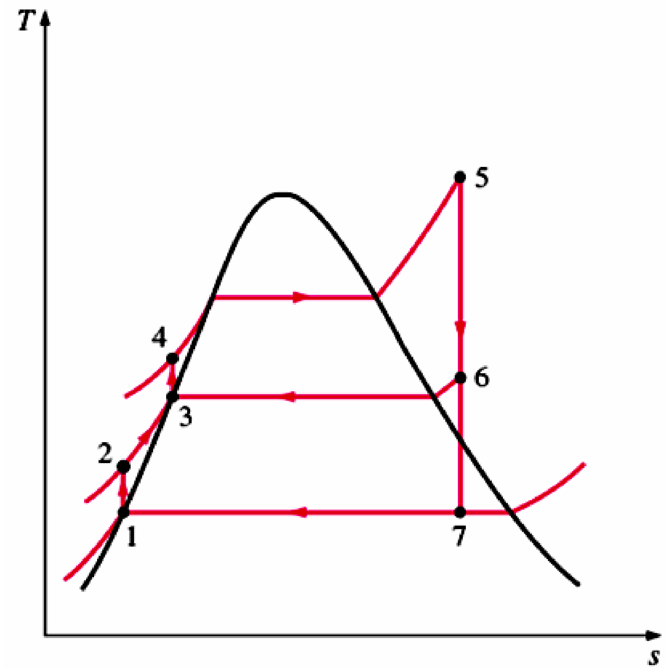




Example

Consider a steam power plant operating on the ideal regenerative Rankine cycle with one open feed water heater. Steam enters the turbine at 15 MPa and 600^{O}C and is condensed in the condenser at a pressure of 10 kPa. Some steam leaves the turbine at a pressure of 1.2 MPa and enters the open feed water heater. Determine the fraction of steam extracted from the turbine and the thermal efficiency of the cycle.



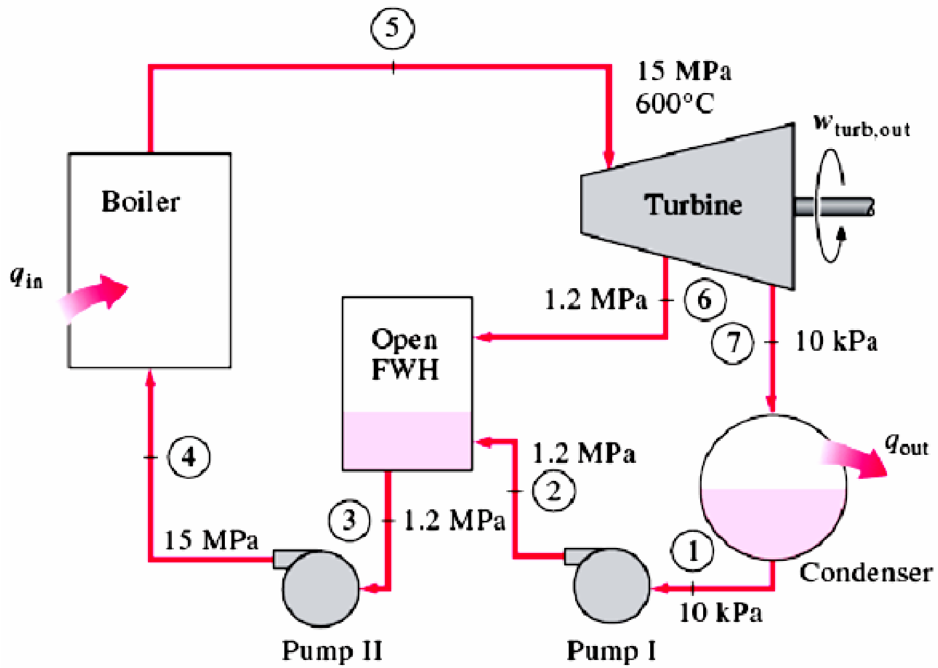


Assumptions

1. Steady operating conditions
2. ΔKE & ΔPE are negligible
3. Ideal Regenerative Rankine cycle

a) Fraction of steam extracted from the turbine

$$y = \frac{h_3 - h_2}{h_6 - h_2}$$



State 1: $P_1 = 10\ kPa$, Sat. liquid $\Big\}$ $h_1 = h_{f@10kPa} = 191.81\ kJ/kg$

$v_1 = v_{f@10kPa} = 0.00101\ \frac{m^3}{kg}$

State 2: $P_2 = 1.2\ MPa$

$s_2 = s_1$

$$w_{pump1,in} = v_1 (P_2 - P_1)$$

$$= (0.00101\ \frac{m^3}{kg})((1200-10)\ kPa)\left(\frac{1\ kJ}{1\ kPa.m^3}\right)$$

$$= 1.2\ kJ/kg$$

$$h_2 = h_1 + w_{pump1,in}$$

$$= 191.8 + 1.2 = 193.01\ kJ/kg$$

State 3: $P_3 = 1.2$ MPa, Sat. liquid $\Big\}$ $v_3 = v_{f@1.2\,MPa} = 0.001138 \ \frac{m^3}{kg}$

$h_3 = h_{f@1.2\,MPa} = 798.33 \ \frac{kJ}{}$

State 4: $P_4 = 15$ MPa

$s_4 = s_3$

$$w_{pump2,in} = v_3 (P_4 - P_3)$$

$$= \left(0.001138 \ \frac{m^3}{kg}\right)\left((1500 - 120)\ kPa\right)\left(\frac{1\ kJ}{1\ kPa \cdot 1}\right.$$

$$= 15.7 \ kJ/kg$$

$$h_4 = h_3 + w_{pump2,in}$$

$$= (798.33 + 15.7) = 814.03 \ kJ/kg$$

State 5: $\left.\begin{array}{l} P_5 = 15 \text{ MPa} \\ T_5 = 600\ ^\circ C \end{array}\right\}$ $\begin{array}{l} h_5 = 3583.1 \text{ kJ/kg} \\ s_5 = 6.6796 \text{ kJ/kg.K} \end{array}$

State 6: $\left.\begin{array}{l} P_6 = 1.2 \text{ MPa} \\ s_6 = s_5 \end{array}\right\}$ $\begin{array}{l} h_6 = 2860.2 \text{ kJ/kg} \\ T_6 = 218.4\ ^\circ C \end{array}$

State 7: $P_7 = 10$ kPa

$s_7 = s_5$

$$x_7 = \frac{s_7 - s_f}{s_{fg}} = \frac{6.6796 - 0.6492}{7.4996} = 0.804$$

$$h_7 = h_f + x_7 h_{fg} = 191.81 + 0.8041\,(2392.1)$$

$$= 2115.3 \text{ kJ/kg}$$

Energy analysis on. feed water heater : assume well insula

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{sys}^{\,0}$$

$$y h_6 + (1-y) h_2 = (1) h_3$$

Recall $y = \frac{\dot{m}_6}{\dot{m}_5}$

Thus $$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{798.33 - 193.01}{2860.2 - 193.01} = 0.227$$

Therefore

$$q_{in} = h_5 - h_4 = (3583.1 - 814.03) = 2769.1 \text{ kJ/kg}$$

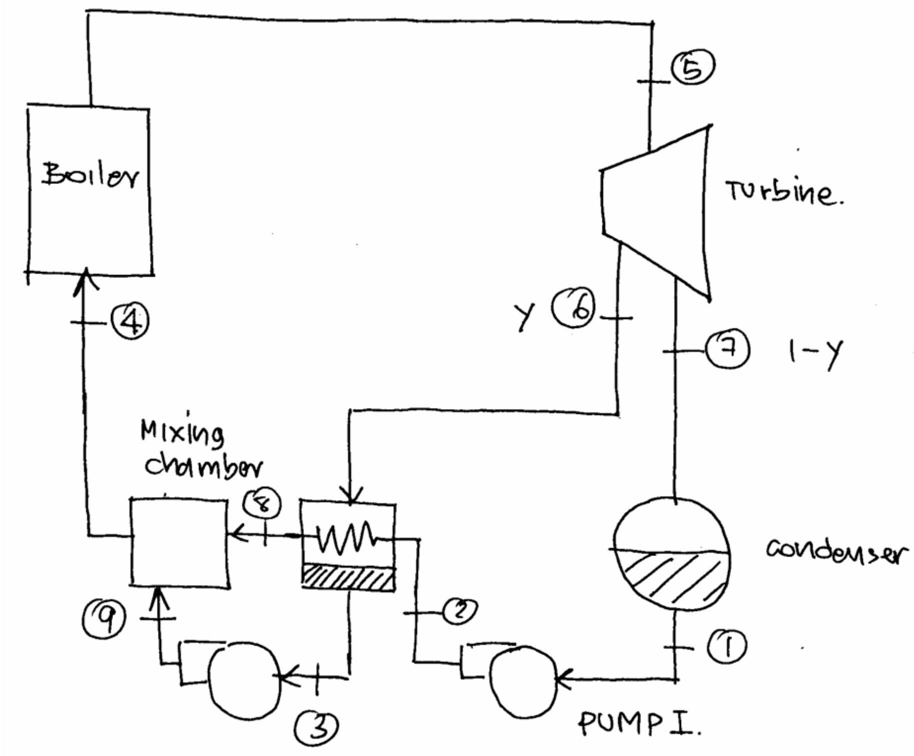
$$q_{out} = (1-y)(h_7 - h_1) = (1 - 0.277)(2115.3 - 191.8)$$

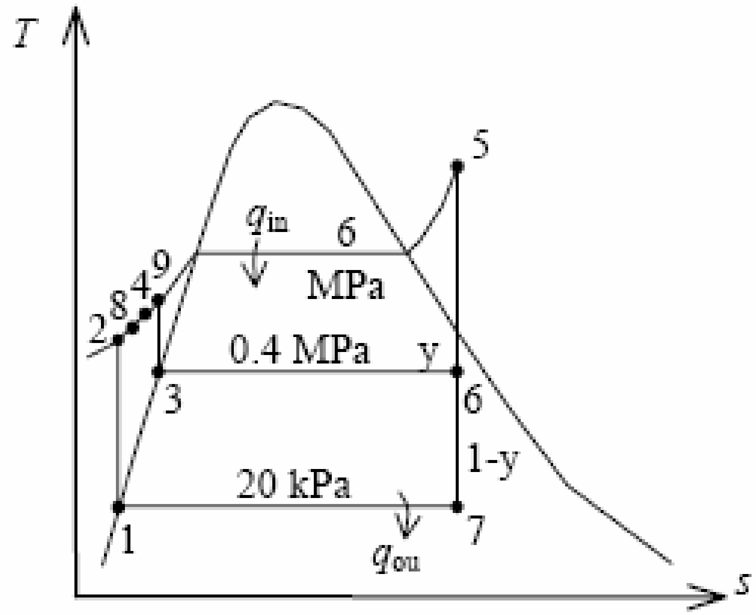
$$= 1486.9 \text{ kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_{in}}{q_{out}} = 1 - \frac{1486.9}{2769.1} = 0.463 \text{ or. } 46.3\%$$

Example: Closed feedwater heater

A steam power plant operates on an ideal regenerative Rankine cycle. Steam enters the turbine at 6 MPa 450^{O}C and is condensed in the condenser at 20 kPa. Steam is extracted from the turbine at 0.4 MPa to heat the feedwater in a closed feedwater heater. Water leaves the heater at the condensation temperature of the extracted steam and that the extracted steam leaves the heater as a saturated liquid and is pumped to the line carrying the feedwater. Determine (a) the net work output per kilogram of steam flowing throught the boiler and (b) the thermal efficiency of the cycle.





Assumptions

1. Steady operating condition exist
2. Δke & Δpe are negligible

From. steam table

$$h_1 = h_{f@20kPa} = 251.42 \ kJ/kg$$

$$v_1 = v_{f@20kPa} = 0.001017 \ m^3/kg$$

$$w_{p,I,in} = v_1 (P_1 - P_2)$$

$$= (0.001017 \ \frac{m^3}{kg})(6000 - 20 \ kPa)\left(\frac{1kJ}{1kPa.m^3}\right)$$

$$= 6.08 \ kJ/kg$$

$$h_2 = h_1 + w_{p,I,in} = 251.42 + 6.08 = 257.50 \ \frac{kJ}{kg}$$

$$\left.\begin{array}{l} P_3 = 0.4 \text{ MPa} \\ \text{Sat. liquid} \end{array}\right\} \quad \begin{array}{l} h_3 = h_{f@0.4\text{ MPa}} = 604.66 \text{ kJ/kg} \\ v_3 = v_{f@0.4\text{ MPa}} = 0.001084 \ \frac{m^3}{kg} \end{array}$$

$$w_{P,II,in} = v_3 (P_9 - P_3)$$

$$= (0.001084 \ \frac{m^3}{kg})(6000 - 400 \text{ kPa})\left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot m^3}\right)$$

$$= 6.07 \text{ kJ/kg}$$

$$h_9 = h_3 + w_{P,II,in} = 604.66 + 6.07 = 610.73 \ \frac{kJ}{kg}$$

$$h_8 = h_3 + v_3 (P_8 - P_3) = h_9 = 610.73 \ \frac{kJ}{kg}$$

Also $h_4 = h_9 = h_8 = 610.73 \ \frac{kJ}{kg}$ since the two fluid streams which are being mixed have the same enthalpy

$$\left.\begin{array}{l} P_5 = 6\text{ MPa} \\ T_5 = 400^\circ\text{C} \end{array}\right\} \quad \begin{array}{l} h_5 = 3302.9\ \text{kJ/kg} \\ s_5 = 6.7219\ \text{kJ/kg.K} \end{array}$$

$$\left.\begin{array}{l} P_6 = 0.4\text{ MPa} \\ s_6 = s_5 \end{array}\right\} \quad x_6 = \frac{s_6 - s_f}{s_{fg}} = \frac{6.7219 - 1.7765}{5.1191} = 0.9661$$

$$h_6 = h_f + x h_{fg}$$

$$= 604.66 + (0.9661)(2133.4)$$

$$= 2665.7\ \frac{\text{kJ}}{\text{kg}}$$

$$\left.\begin{array}{l} P_7 = 20\text{ kPa} \\ s_7 = s_5 \end{array}\right\} \quad x_7 = \frac{s_7 - s_f}{s_{fg}} = \frac{6.7219 - 0.8320}{7.0752} = 0.8325$$

$$h_7 = h_f + x h_{fg}$$

$$= 251.42 + (0.8325)(2357.5)$$

$$= 2214\ \text{kJ/kg}$$

The fraction of steam extracted is determined from the steady-flow energy balance equation applied to the feedwater heater. Noting that $\dot{Q} \cong \dot{W} \cong \Delta ke \cong \Delta pe \cong 0$

$$\dot{E}_{in} - \dot{E}_{out} = \cancel{\Delta \dot{E}_{sys}}^{\,0}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e$$

$$\dot{m}_2 (h_8 - h_2) = \dot{m}_6 (h_6 - h_3)$$

$$(1-y)(h_8 - h_2) = y(h_6 - h_3)$$

where y is the fraction of steam extracted from the turbine $(= \dot{m}_6/\dot{m}_5)$ Solving for y

$$y = \frac{h_8 - h_2}{(h_6 - h_3) + (h_8 - h_2)} = \frac{610.73 - 257.50}{2665.7 - 604.66 + 610.73 - 257.5}$$

$$= 0.1463$$

then $q_{in} = h_5 - h_4 = 3302.9 - 610.73 = 2692.2 \text{ kJ/kg}$

$q_{out} = (1-y)(h_7 - h_1) = (1 - 0.1463)(2214 - 251.42)$

$= 1675.4 \text{ kJ/kg}$

And $w_{net} = q_{in} - q_{out} = 2692.2 - 1675.4$

$= 1016.8 \text{ kJ/kg}$

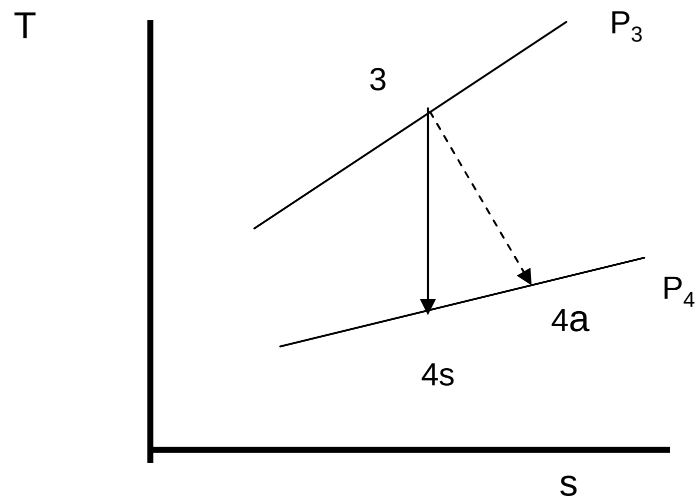
(b) The thermal efficiency is determined from

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1675.4 \text{ kJ/kg}}{2692.2 \text{ kJ/kg}} = 37.8\ \%$$

Deviation from Actual Cycles

- Piping losses--frictional effects reduce the available energy content of the steam.
- Turbine losses--turbine isentropic (or adiabatic) efficiency.

$$\eta_{turb} = \frac{w_{actual}}{w_{isentropic}} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

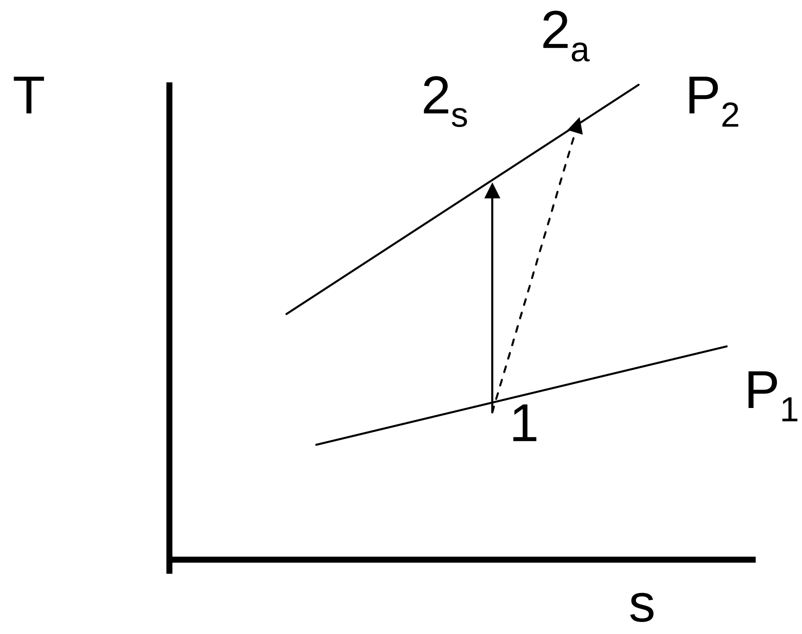


The actual enthalpy at the turbine exit (needed for the energy analysis of the next component) is

$$h_{4a} = h_3 - \eta_{turb}(h_3 - h_{4s})$$

•Pump losses--pump isentropic (or adiabatic) efficiency.

$$\eta_{pump} = \frac{w_{isentropic}}{w_{actual}} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$



The actual enthalpy at the pump exit (needed for the energy analysis of the next component) is

$$h_{2a} = h_1 + \frac{1}{\eta_{pump}}(h_{2s} - h_1)$$

•Condenser losses--relatively small losses that result from cooling the condensate below the saturation temperature in the condenser.

Second law analysis of vapor power cycles

- Ideal Carnot cycle is a totally reversible
- Ideal Rankine cycles may involve irreversibilities external to the system, e.g. heat transfer through a finite temperature difference
- Second law analysis of these cycles is used to reveal where the largest irreversibilities occur and what their magnitude are.

$$\dot{X}_{destruction} = T_0 \dot{S}_{gen} = T_0 \left(\dot{S}_{out} - \dot{S}_{in} \right)$$

$$= T_0 \left(\sum_{out} \dot{m}s + \frac{\dot{Q}_{out}}{T_{b,out}} - \sum_{in} \dot{m}s - \frac{\dot{Q}_{in}}{T_{b,in}} \right)$$

Or on a unit mass basis for a one-inlet, one-exit, steady flow device

$$x_{destruction} = T_0 s_{gen} = T_0 \left(s_e - s_i + \frac{q_{out}}{T_{b,out}} - \frac{q_{in}}{T_{b,in}} \right)$$

Second law analysis of vapor power cycles

- Exergy destruction associated with a cycle depends on the magnitude of the heat transfer with the high- and low- temperature reservoirs.

$$x_{destruction} = T_0 \left(\sum \frac{q_{out}}{T_{b,out}} - \sum \frac{q_{in}}{T_{b,in}} \right)$$

For a cycle that involves heat transfer only with a source at T_H and a sink at T_L, the exergy destruction are

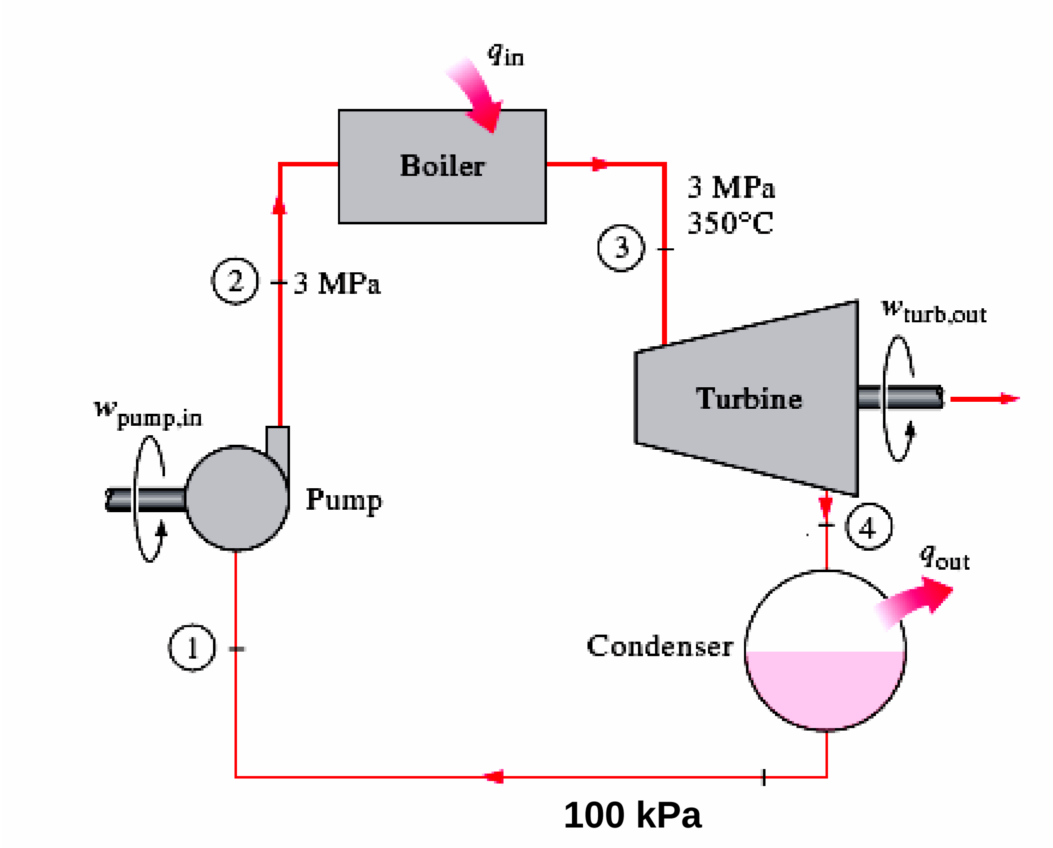
$$x_{destruction} = T_0 \left(\frac{q_{out}}{T_L} - \frac{q_{in}}{T_H} \right)$$

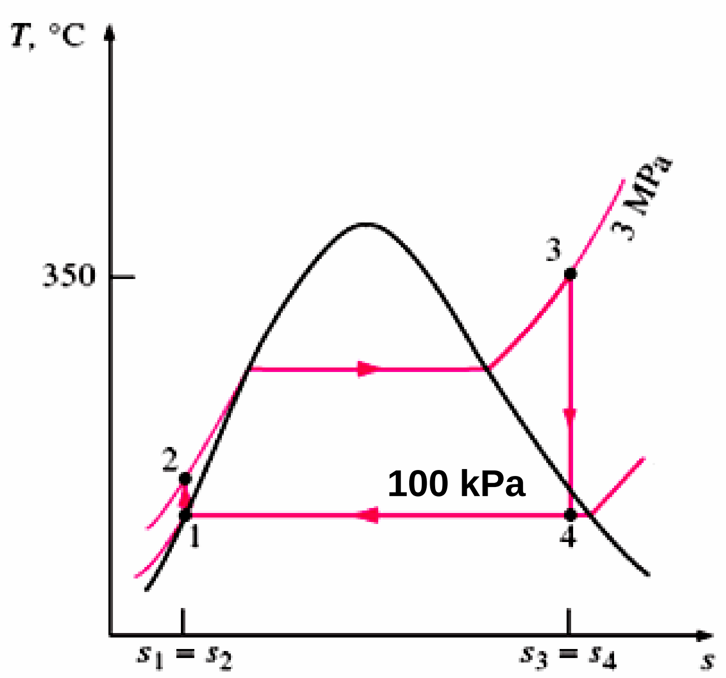
Exergy of a fluid stream ϕ at any states is

$$\varphi = \left(h - h_0\right) - T_0\left(s - s_0\right) + \frac{V^2}{2} + gz$$

Example

Determine the exergy destruction associated with the Rankine cycle (all four processes as well as the cycle), assuming that heat is transferred to the steam in a furnace at 1600 K and heat is rejected to a cooling medium at 290 K and 100 kPa. Also, determine the exergy of the steam leaving the turbine

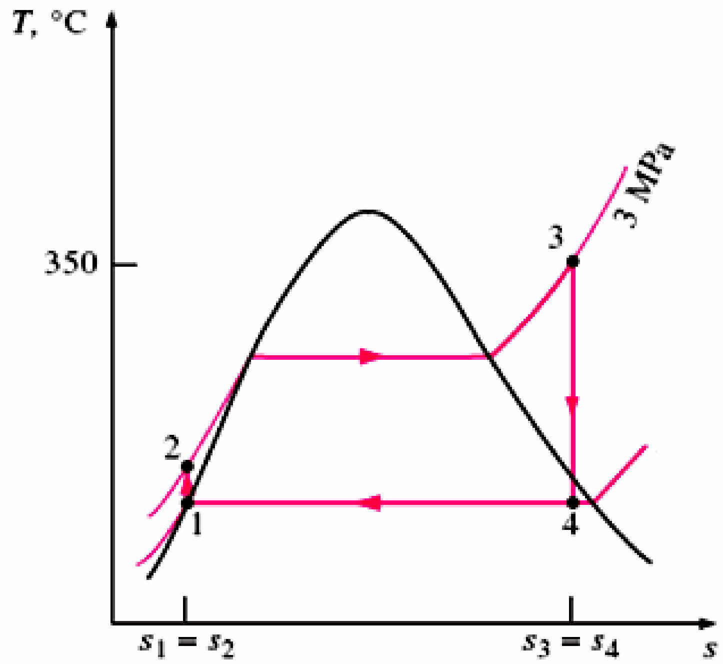




$q_{in} = 2728.6 \ kJ/kg$

$q_{out} = 2018.6 \ kJ/kg$

Process 1-2 & 3-4 are isentropic



$x_{destruction,12} = 0$

$x_{destruction,34} = 0$

Process 2-3 & 4-1 are constant pressure

(heat addition) (heat rejection)

from. steam table

$s_2 = s_1 = s_{f@75 kPa} = 1.2132 \ kJ/kg.K$

$s_4 = s_3 = 6.745 \ kJ/kg.K$ (at 3 MPa, 350°C)

Thus.

$$x_{destruction,23} = T_0\left(s_3 - s_2 - \frac{q_{in,23}}{T_{source}}\right)$$

$$= (290k)\left\{(6.745 - 1.2132) - \frac{2928.6\ kJ/kg}{1600\ k}\right.$$

$$= 1110\ kJ/kg$$

$$x_{destruction,41} = T_0\left(s_1 - s_4 + \frac{q_{out,41}}{T_{sink}}\right)$$

$$= (290\ k)\left\{(1.2132 - 6.7450) + \frac{2018.6\ kJ/kg}{290\ k}\right)$$

$$= 414\ kJ/kg$$

Therefore, the irreversibility of the cycle is $x_{destruct,cycle}$

$$= x_{des,12} + x_{des,23} + x_{des,34} + x_{des,41}$$

$$= 0 + 1110 + 0 + 414 = 1524\ kJ/kg$$

$$\text{Exergy, } \psi_4 = (h_4 - h_0) - T_0(s_4 - s_0) + \cancel{\frac{V_4^2}{2}}^{\,0} + \cancel{g z_4}^{\,0}$$

$$= (h_4 - h_0) - T_0(s_4 - s_0)$$

$$h_0 = h_{@290K,\,100\,kPa} \cong h_{f@290K} = 71.355 \text{ kJ/kg}$$

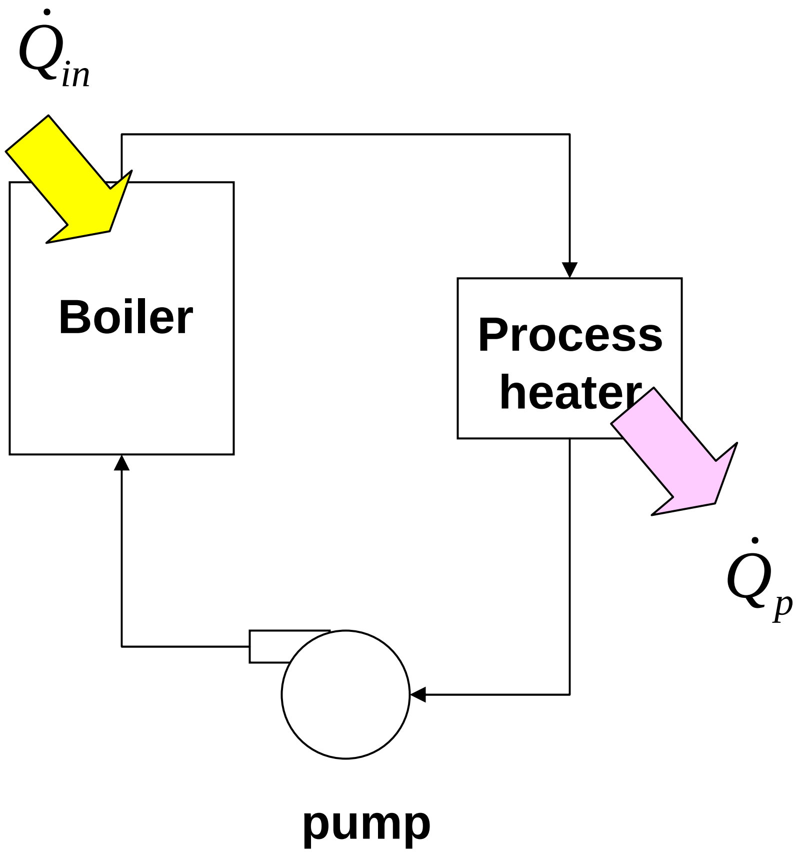
$$s_0 = s_{@290K,\,100\,kPa} \cong s_{f@290K} = 0.2533 \text{ kJ/kg.K}$$

Thus

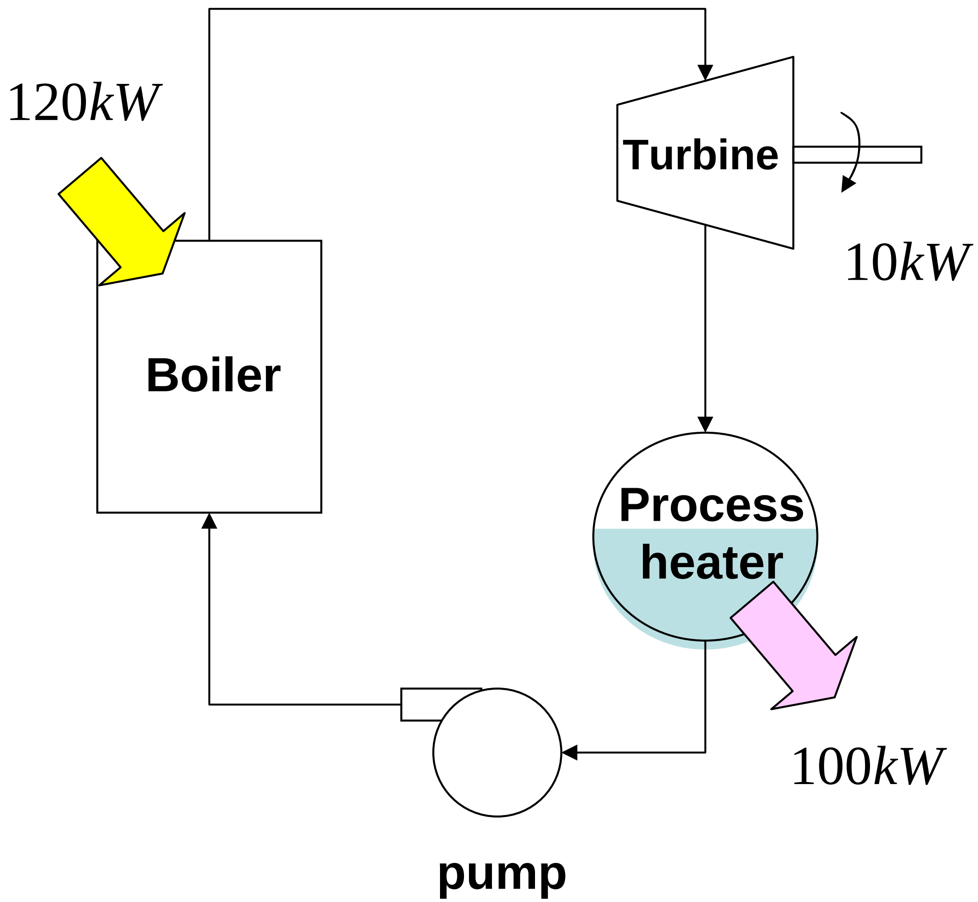
$$\psi_4 = (2403 - 71.355) - (290)\{6.745 - 0.2533\}$$

Cogeneration

Cogeneration is the production of more than one useful form of energy (such as process heat and electric power) from the same energy source



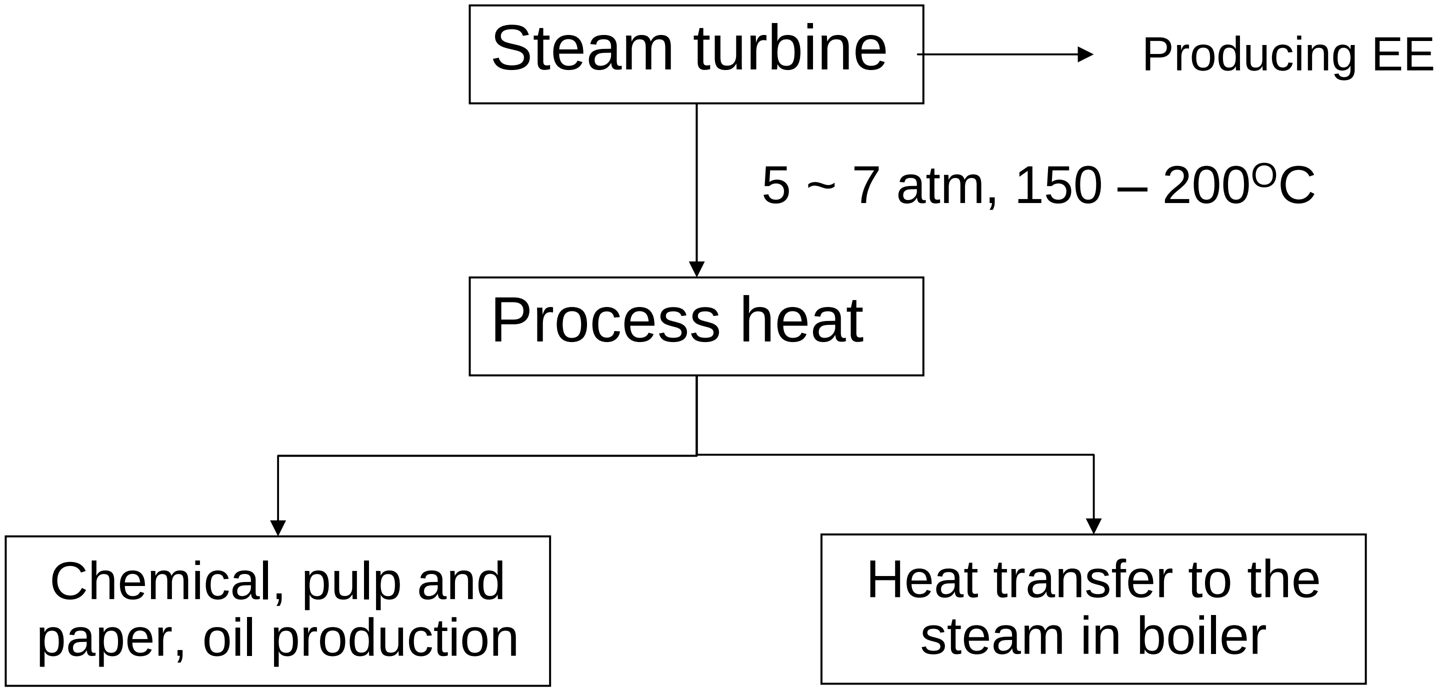
A simple process-heating plant



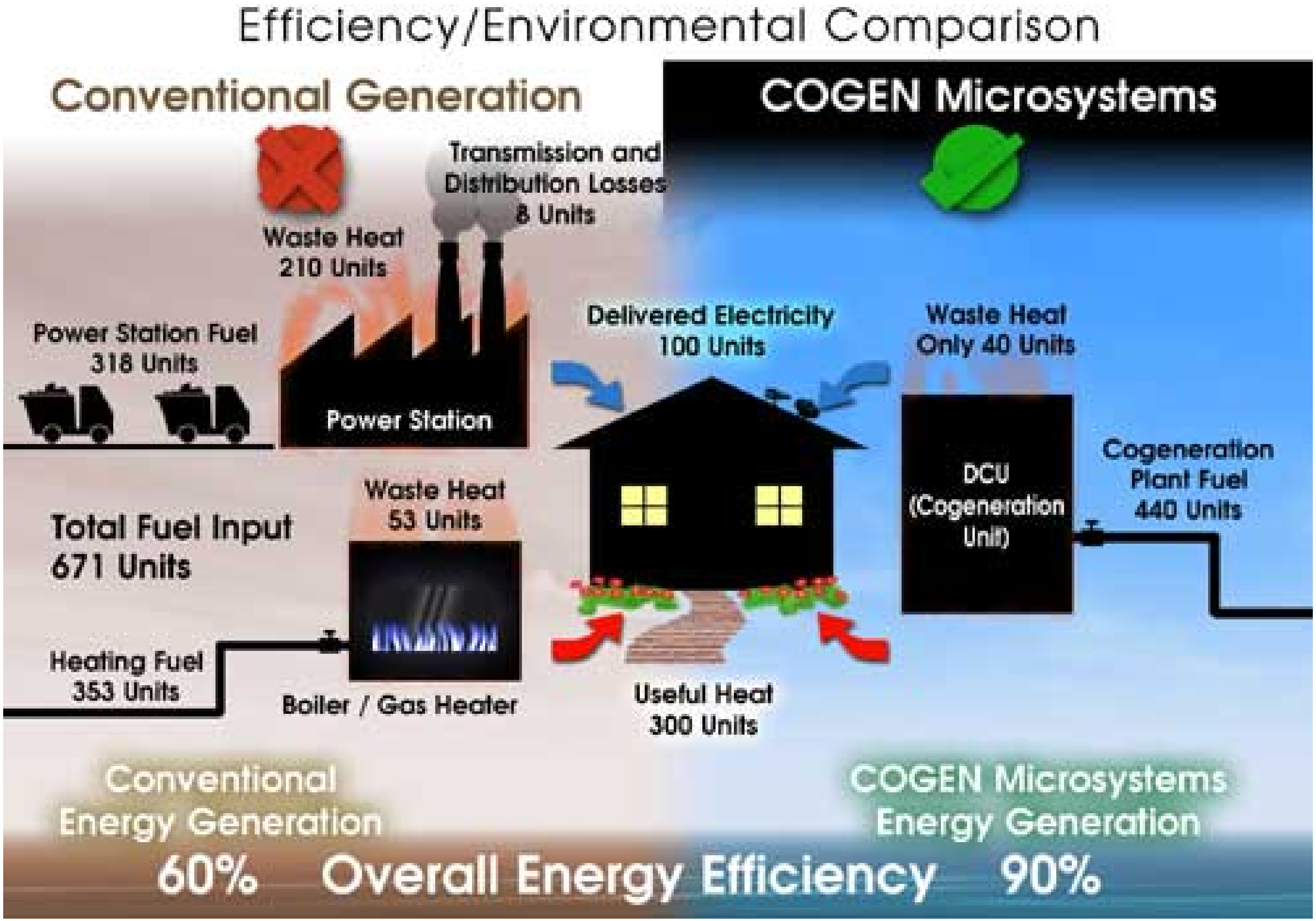
An ideal cogeneration plant

Cogeneration

Cogeneration is the production of more than one useful form of energy (such as process heat and electric power) from the same energy source

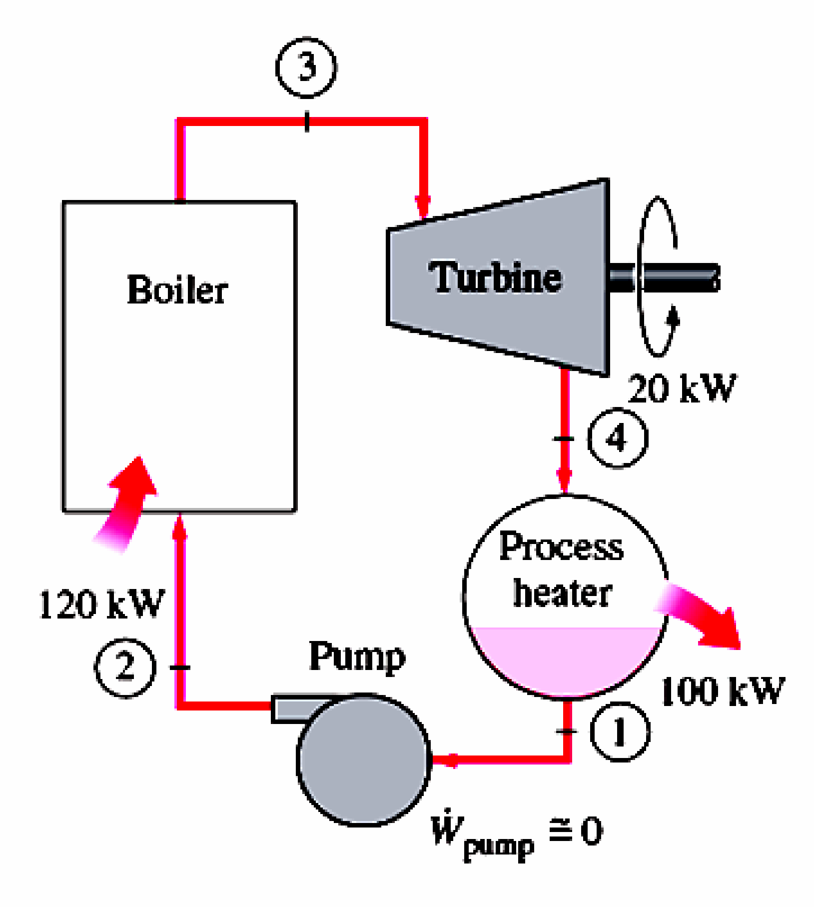


Cogeneration



Utilization factor, ε_u

Waste heat rejection from turbine transfers to the steam in boiler and is utilized as either process heat or electric power.



Utilization factor, ε_u

$$\varepsilon_u = \frac{\text{Net work output} + \text{Process heat delivered}}{\text{Total heat input}}$$

$$= \frac{\dot{W}_{net} + \dot{Q}_p}{\dot{Q}_{in}}$$

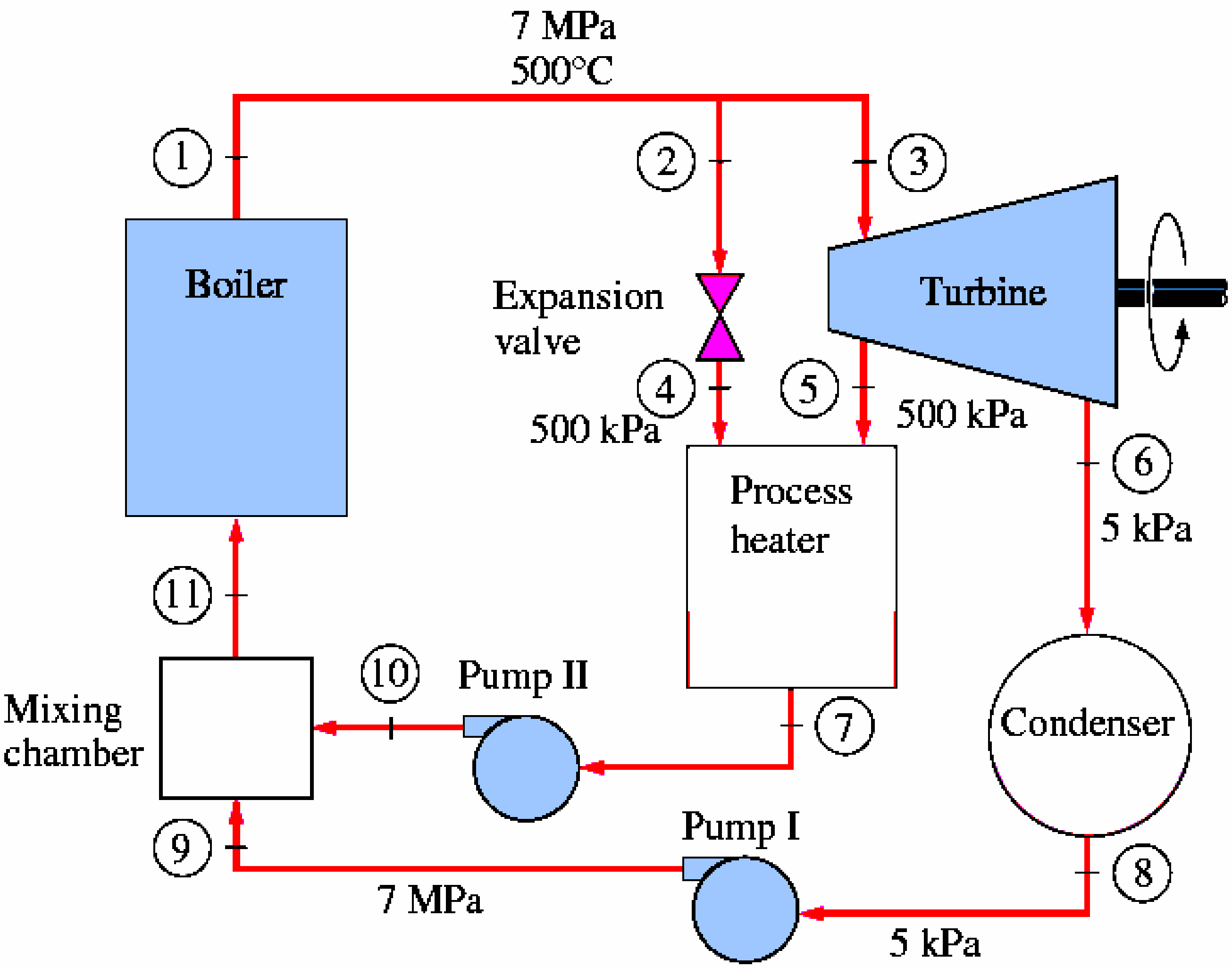
$$\boxed{\varepsilon_u = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}}$$

$\dot{Q}_{out}$ = presents the heat rejected in the condenser.

ε_u = 100% when no any heat losses in system

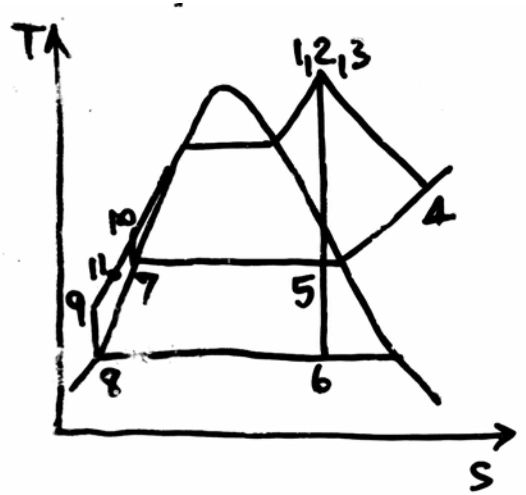
Example

Steam enters the turbine at 7 MPa and 500^{O}C. Some steam is extracted from the turbine at 500 kPa for process heating. The remaining steam continues to expand to 5 kPa. Steam is then condensed at constant pressure and pumped to the boiler pressure of 7 MPa. At times of high demand for process heat, some steam leaving the boiler is throttled to 500 kPa and is routed to the process heater. The extraction fraction are adjusted so that steam leaves the process heater as a saturated liquid at 500 kPa. It is subsequently pumped to 7 MPa. The mass flow rate of steam through the boiler is 15 kg/s. Disregarding any pressure drops and heat losses in the piping and assuming the turbine and the pump to be isentropic, determine (a) the maximum rate at which process heat can be supplied, (b) the power produced and the utilization factor when no process heat is supplied, and (c) the rate of process heat supply when 10 percent of the steam is extracted before it enters the turbine and 70 percent of the steam is extracted from the turbine at 500 kPa for process heating.


7 MPa
500°C
1
2
3
Boiler
Expansion
valve
Turbine
4
5
500 kPa
500 kPa
Process
heater
6
5 kPa
11
10
Pump II
Mixing
chamber
7
Condenser
9
Pump I
8
7 MPa
5 kPa

Assumptions

1. Steady operating conditions
2. Pressure drops and heat losses in piping are negligible
3. ΔKE & ΔPE are negligible.



$$w_{pump1,in} = v_9 (P_9 - P_8)$$

$$= \left(0.001005 \frac{m^3}{kg}\right)\left\{(7000-5)\ kPa\right\}\left(\frac{1\ kJ}{1\ kPa \cdot m^3}\right)$$

$$= 7.03 \ \frac{kJ}{kg}$$

$$w_{pump2,in} = v_7 (P_{10} - P_7)$$

$$= \left(0.001093 \frac{m^3}{kg}\right)\left\{(7000-500)\ kPa\right\}\left(\frac{1\ kJ}{1\ kPa \cdot m^3}\right)$$

$$= 7.10 \ kJ/kg$$

$$h_1 = h_2 = h_3 = h_4 = 3411.4 \ kJ/kg$$

$h_6 = 2073 \text{ kJ/kg}$

$h_7 = h_{f@500\text{ kPa}} = 640.09 \text{ kJ/kg}$

$h_8 = h_{f@5\text{ kPa}} = 137.75 \text{ kJ/kg}$

$h_9 = h_8 + w_{pump1,in} = (137.75 + 7.03)$

$= 144.78 \text{ kJ/kg}$

$h_{10} = h_7 + w_{pump2,in} = 640.09 + 7.10$

$= 647.19 \text{ kJ/kg}$

(a) Max. rate of process heat is when all the steam leaving the boiler is throttled and sent to the process heater and none is sent to the turbine ($\dot{m}_4 = \dot{m}_7 = \dot{m}_1 = 15$ kg/s and $\dot{m}_3 = \dot{m}_5 = \dot{m}_6 = 0$)

$$\dot{Q}_{p,max} = \dot{m}_1 (h_4 - h_7)$$

$$= (15 \text{ kg/s})(3411.4 - 640.9)\ \frac{\text{kJ}}{\text{kg}} = 41{,}570 \text{ kW}$$

$\varepsilon_u = 100\%$ (no heat is rejected in the condenser, heat loss from the piping and other components are negligib and combustion losses are not considered)

(b) When no process heat is supplied, all the steam leaving the boiler passes through the turbine and expands to the condenser pressure of 5 kPa ($\dot{m}_3 = \dot{m}_6 = \dot{m}_1 = 15\ kg/s$ and $\dot{m}_2 = \dot{m}_5 = 0$)

$$\dot{W}_{turb,out} = \dot{m}(h_3 - h_6) = (15\ kg/s)\left\{(3411.4 - 2073)\ \frac{kJ}{kg}\right\}$$
$$= 20{,}076\ kW$$

$$\dot{W}_{pump,in} = (15\ \frac{kg}{s})(7.03\ \frac{kJ}{kg}) = 105\ kW$$

$$\dot{W}_{net,out} = \dot{W}_{turb,out} - \dot{W}_{pump,in}$$

$$= 20{,}076 - 105 = 19{,}971\ kW \cong 20\ MW$$

$$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_{11})$$

$$= (15 \ \tfrac{kg}{s}) \left\{ (3411.4 - 144.78) \ \tfrac{kJ}{kg} \right\}$$

$$= 48.999 \ kW$$

Thus.

$$\epsilon_u = \frac{\dot{W}_{net} + \dot{Q}_p}{\dot{Q}_{in}} = \frac{19971 + 0}{48999}$$

$$= 0.408 \text{ or. } 40.8\%$$

(c) Neglect any ΔKE & ΔPE

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_4 h_4 + \dot{m}_5 h_5 = \dot{Q}_{p,out} + \dot{m}_7 h_7$$

$$\dot{Q}_{p,out} = \dot{m}_4 h_4 + \dot{m}_5 h_5 - \dot{m}_7 h_7$$

$$\dot{m}_4 = (0.1)(15 \text{ kg/s}) = 1.5 \text{ kg/s}$$

$$\dot{m}_5 = (0.7)(15 \text{ kg/s}) = 10.5 \text{ kg/s}$$

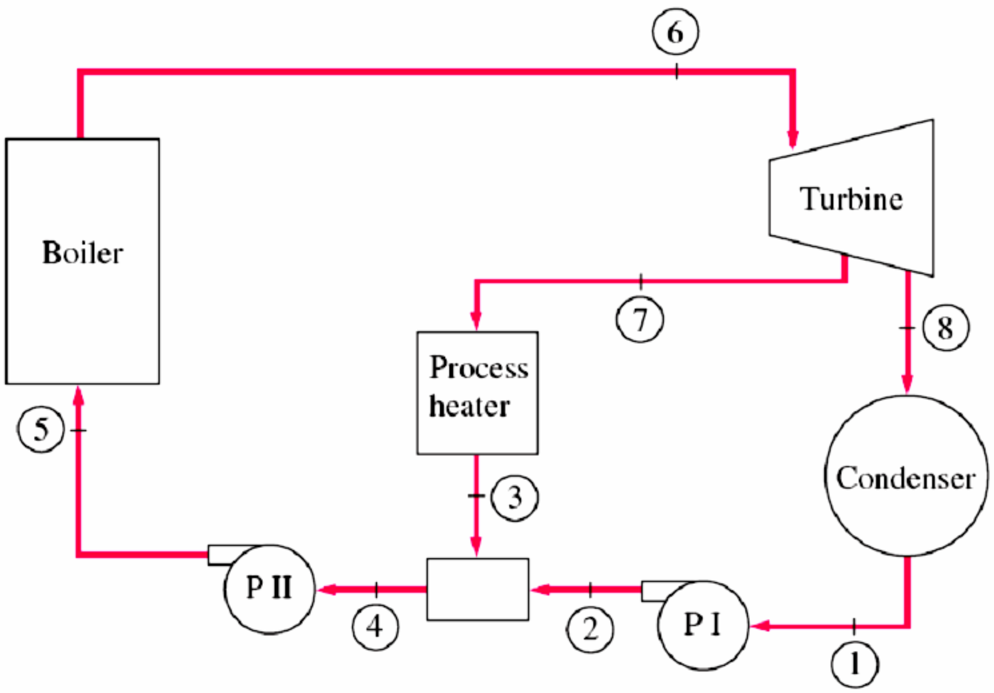
$$\dot{m}_7 = \dot{m}_4 + \dot{m}_5 = 1.5 + 10.5 = 12 \text{ kg/s}$$

Thus

$$\dot{Q}_{p,out} = (1.5\ kg/s)(3411.4\ kJ/kg)$$
$$+ (10.5\ kg/s)(2939.3\ kJ/kg)$$
$$- (12\ kg/s)(640.09\ kJ/kg)$$
$$= 26.2\ MW$$ ← use in process heater

Example

A textile plane require 4 kg/s of saturated steam at 2 MPa, which is extracted from the turbine of a cogeneration plant. Steam enters the turbine at 8 MPa and 500^{O}C at a rate of 11 kg/s and leaves at 20 kPa. The extracted steam leaves the process heater as a saturated liquid and mixes with the feed water at constant pressure. The mixture is pumped to the boiler pressure. Assuming and isentropic efficiency of 88 percent for both the turbine and the pumps, determine (a) the rate of process heat supply, (b) the net power output, and (c) the utilization factor of the plant.



Assumptions

1. Steady operating conditions
2. ΔKE & ΔPE are negligible

$h_1 = h_{f@20kPa} = 251.42$ kJ/kg

$v_1 = v_{f@20kPa} = 0.001017$ m^3/kg

$$w_{pump\,I,in} = \frac{v_1(P_2 - P_1)}{\eta_p}$$

$$= \frac{(0.001017 \frac{m^3}{kg})(2000-20)\,kPa}{(0.88)}\left(\frac{1\,kJ}{1\,kPa.m^3}\right)$$

$$= 2.29 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{pump\,1,\dot{m}}$$

$$= 251.42 + 2.29 = 253.71 \ \frac{kJ}{kg}$$

$$h_3 = h_{f@2MPa} = 908.47 \ kJ/kg$$

Mixing chamber

$$\dot{m}_3 h_3 + \dot{m}_2 h_2 = \dot{m}_4 h_4$$

$$\left(4 \frac{kg}{s}\right)\left(908.47 \frac{kJ}{kg}\right) + \left(11-4 \frac{kg}{s}\right)\left(253.71 \frac{kJ}{kg}\right) = \left(11 \frac{kg}{s}\right) h_4$$

$$h_4 = 491.81 \ \frac{kJ}{}$$

$$v_4 \cong v_{f@h_f=491.81 \frac{kJ}{kg}} = 0.001058 \ \frac{m^3}{kg}$$

$$w_{pump2,in} = \frac{v_4 (P_5 - P_4)}{\eta_p}$$

$$= (0.001058 \ \frac{m^3}{kg}) (\frac{8000 - 2000 \ kPa}{0.88}) (\frac{1 \ kJ}{1 \ kPa . m^3})$$

$$= 7.21 \ kJ/kg$$

$$h_5 = h_4 + w_{pump2,in}$$

$$= 491.81 + 7.21 = 499.02 \ \frac{kJ}{kg}$$

$$\left. \begin{array}{l} P_6 = 8 \ MPa \\ T_6 = 500 \ ^\circ C \end{array} \right\} \begin{array}{l} h_6 = 3399.5 \ kJ/kg \\ s_6 = 6.7266 \ kJ/kg.K \end{array}$$

$$\left. \begin{array}{l} P_7 = 2 \ MPa \\ s_7 = s_6 \end{array} \right\} h_{7s} = 3000.4 \ \frac{kJ}{kg}$$

$$\eta_T = \frac{h_6 - h_7}{h_6 - h_{7s}} \longrightarrow h_7 = h_6 - \eta_T (h_6 - h_{7s})$$

$$= 3399.5 - (0.88)(3399.5 - 3000.4)$$

$$= 3048.3 \ \text{kJ/kg}$$

$$\eta_T = \frac{h_6 - h_8}{h_6 - h_{8s}} \longrightarrow h_8 = h_6 - \eta_T (h_6 - h_{8s})$$

$$= 3399.5 - (0.88)(3399.5 - 2215.5)$$

$$= 2357.6 \ \frac{\text{kJ}}{\text{kg}}$$

Then. $\dot{Q}_p = \dot{m}_7 (h_7 - h_3)$

(b) $\dot{W}_{T,out} = \dot{m}_7 (h_6 - h_7) + \dot{m}_8 (h_6 - h_8)$

$$= (4)(3399.5 - 3048.3)$$

$$+ (7)(3399.5 - 2357.6)$$

$$= 8698 \ \text{kW}$$

$$\dot{W}_{pump,m} = \dot{m}_1 w_{pump1,m} + \dot{m}_4 w_{pump2,m}$$

$$= (7)(2.29) + (11)(7.21)$$

$$= 95 \ \text{kW}$$

$$\dot{W}_{net} = \dot{W}_{T,out} - \dot{W}_{pump,m}$$

$$= 8698 - 95$$

$$= 8603 \ \text{kW}$$

(c)

$$\dot{Q}_{in} = \dot{m}_5 (h_6 - h_5)$$

$$= (11)(3399.5 - 499.02)$$

$$= 31{,}905 \ \text{kW}$$

Utilization factor, $\varepsilon_u = \dfrac{\dot{W}_{net} + \dot{Q}_p}{\dot{Q}_{in}}$

$$= \frac{8603 + 8559}{31905}$$

$$= 0.538 \text{ or } 53.8\%$$